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PROCEEDINGS

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VOL. 74 MAY, 1948 No. 5

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AND

DISCUSSIONS

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DISCUSSIONS

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GUIDEPOST FOR TECHNICAL READERS
MAY, 1948, *PROCEEDINGS*

Items of value or significance to readers in various fields are here listed. For convenience, the distribution is given in terms of the Society's Technical Divisions

(A constant effort is made to supply technical material to Society members, over the entire range of possible interest. In so far as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement.—Ed.)

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

THE RELATIONSHIP BETWEEN PILE FORMULAS AND LOAD TESTS

By Robert D. Chellis,[1] M. ASCE

Synopsis

Pile-driving formulas are often used as the sole criterion of pile bearing capacity, without load tests. When load tests have been made, the results often have been at variance with the computed values. Consideration of the physical properties of soils and piles, and of the interpretations of terms in the formulas, indicates the reasons for such discrepancies, and also reliable means of avoiding them. Driving resistances bear definite relationship to static load resistances only in soils in which the predominating resistance is supplied by cohesionless materials. A study of forty-five cases in soils of this type, in which test load failure values and resistances computed from various formulas in use are compared, indicates the unsatisfactory spread of values obtained from the most common abbreviated formulas and the generally satisfactory results from the more comprehensive formulas. Use of a static formula for preliminary determination of pile lengths and as a check in all other cases appears advisable.

Introduction

Is there any relation between bearing capacity of a pile and its driving resistance? This question has long caused controversy. Scores of dynamic, static, and empirical formulas have been offered.

Dynamic formulas attempt to equate the kinetic energy of the hammer blow to the set (the amount of penetration of the pile tip under one blow of the hammer), or resistance of the soil to penetration. Static formulas equate the load-carrying capacity of the pile to the bearing and shearing values of the soil or to friction between pile and soil. Empirical formulas are usually based on the results of tests for limited conditions, or on the strength of the pile as a post without relation to driving or soil conditions. The formulas in general use are of the dynamic type.

Note—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1948.

[1] Structural Engr., Stone & Webster Eng. Corp., Boston, Mass.

This paper does not purport to derive the various formulas, nor to analyze their mathematical merits. The formulas may be considered as theoretical or empirical in fact, if desired. It is the object of the paper to perform the service of comparing the results given by the most common formulas against those obtained from load tests. This will permit forming some opinion as to the spread of values which may be expected, and as to the actual factors of safety which may result.

Dynamic Formulas

The dynamic formula most commonly employed up to the present in the United States is the so-called "Engineering-News" formula. This formula received its name from the fact that it was proposed more than half a century ago by an editor of that magazine and appeared in its pages.[2] The Eytelwein formula often has been used with heavy piles. Modifications of the Engineering-News and the Eytelwein formulas, and the Navy-McKay formula have often been recommended for use with heavy piles. These modifications consist of increasing the last term in the denominator.

In England, the Hiley formula[3] is specified by the Institution of Structural Engineers, in London, and is in common use, although with the value of L considered as the full length of the pile.[4] It is also specified in slightly modified form in the Canadian National Building Code. Another modification of this formula is given in the Pacific Coast Uniform Building Code, and it is also specified in some form in other codes.

For convenience and comparison these formulas are now shown together. The symbols used are defined in the Appendix.

The Engineering-News formula, for drop hammers, is

$$R = \frac{2\,W_r\,H}{s + 1.0} \qquad (1)$$

for single-acting steam hammers,

$$R = \frac{2\,W_r\,H}{s + 0.1} \qquad (2)$$

and, for double-acting and differential-acting steam hammers,

$$R = \frac{2\,E_n}{s + 0.1} \qquad (3)$$

Eq. 3 has come to be used with double-acting hammers, not on the basis of reasoning, but because it was readily available and simple. Mathematically, Eqs. 1, 2, and 3 are based on inclusion of a safety factor of 6.

The Eytelwein formula is, for drop hammers,

$$R = \frac{2\,W_r\,H}{s\left(1 + \frac{W_p}{W_r}\right)} \qquad (4)$$

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[2] "Formulae for Safe Loads of Bearing Piles," by A. M. Wellington, *Engineering News*, December 29, 1888, pp. 509–512.

[3] "Pile Driving Calculations with Notes on Driving Forces and Ground Resistance," by A. Hiley, *The Structural Engineering*, July and August, 1930, pp. 246–259.

[4] "Specification for Concrete Pile-Driving," The Institution of Structural Engineers, London, 1936.

For single-acting steam hammers, Eq. 4 becomes

$$R = \frac{2\,W_r\,H}{s + 0.1\,\dfrac{W_p}{W_r}} \qquad (5)$$

Mathematically, Eqs. 4 and 5 also are based on inclusion of a safety factor of 6.

TABLE 1.—ALLOWANCE FOR TEMPORARY COMPRESSION FOR HEAD OF PILE AND PILE CAP, C_1, IN INCHES

Description	STRESS ON DRIVING CAP (LB PER SQ IN.)			
	500	1,000	1,500	2,000
Head of wood pile	0.05	0.10	0.15	0.20
Pre-cast concrete pile, 3-in. to 4-in. packing in cap	0.05[a] +0.07[b]	0.10[a] +0.15[b]	0.15[a] +0.22[b]	0.20[a] +0.30[b]
Pre-cast concrete pile, ½-in. to 1-in. pad on pile	0.025	0.05	0.075	0.10
Steel pile, wood packing with steel cover	0.04	0.08	0.12	0.16
Fiber disk $\frac{3}{16}$ in. thick between steel plates on steel pile	0.02	0.04	0.06	0.08
Head of steel pile	0	0	0	0

[a] Compression of cap and dolly above. [b] For packing below cap.

The Hiley formula, for drop hammers and single-acting steam hammers, is

$$R_u = \frac{e_f\,W_r\,h}{s + \frac{1}{2}(C_1 + C_2 + C_3)} \times \frac{W_r + e^2\,W_p}{W_r + W_p} \qquad (6)$$

and, for double and differential-acting steam hammers,

$$R_u = \frac{12\,e_f\,E_n}{s + \frac{1}{2}(C_1 + C_2 + C_3)} \times \frac{W_r + e^2\,W_p}{W_r + W_p} \qquad (7)$$

in which

$$C_2 = \frac{R_u\,l}{A\,E} \qquad (8)$$

and values for C_1 and C_3 are given in Tables 1 and 2, respectively.

The Pacific Coast Uniform Building Code formula is

$$R_u = \frac{12\,W_r\,H\,\dfrac{W_r + K\,W_p}{W_r + W_p}}{s + \dfrac{12\,R_u\,L}{A\,E}} \qquad (9)$$

in which $K = 0.25$ for steel piles and 0.10 for other piles. A safety factor of 4 is to be applied to the result to obtain the safe working load.

TABLE 2.—TEMPORARY COMPRESSION ALLOWANCE[a] FOR QUAKE OF GROUND, C_3, IN INCHES

STRESS ON TIP[b] (LB PER SQ IN.)			
500	1,000	1,500	2,000
0 to 0.10	0.10 to 0.20	0.10 to 0.30	0.05 to 0.20

[a] These values may be double if the strata underlying the pile tips are very soft. [b] Cross section of bounded area for H-pile.

The Canadian National Building Code formula is

$$R = \frac{4\,W_r\,H\,n}{s + (\frac{1}{2})\,C} \qquad (10a)$$

in which, for friction piles,

$$n = \frac{W_r + e^2 W_p}{W_r + W_p} \qquad (10b)$$

for refusal,

$$n = \frac{W_r + 0.5\, e^2 W_p}{W_r + W_p} \qquad (10c)$$

and

$$C = \frac{3\,R}{A}\left(\frac{l}{E} + 0.0001\right) \qquad (10d)$$

The resulting value is for drop hammers with triggers and should be multiplied by 0.9 for single-acting steam hammers, and 0.8 for drop hammers with winch drag. Eq. 10*a* gives safe working loads, predicated upon a safety factor of 3.

The modified Engineering-News formula, for single and double-acting hammers only, is

$$R = \frac{2\,W_r\,H}{s + 0.3} \qquad (11)$$

Mathematically, Eq. 11 is based on inclusion of a safety factor of 6.

The modified Eytelwein formula, for single and double-acting hammers only, is

$$R = \frac{2\,W_r\,H}{s + 0.3\,\frac{W_p}{W_r}} \qquad (12)$$

Eq. 12 is based on inclusion of a safety factor of 6.

The Navy-McKay formula, also for single and double-acting hammers only, is

$$R = \frac{2\,W_r\,H}{s\left(1 + 0.3\,\frac{W_p}{W_r}\right)} \qquad (13)$$

Eq. 13 is likewise based on inclusion of a safety factor of 6.

The Engineering-News formula was developed in the days when all piles were of timber and all were driven with a drop hammer. With the development of single-acting steam hammers an empirical modification was introduced, stated to take account of the soil lubricating action caused by more rapid strokes. It should be noted that this empirical change which affects the answer considerably was made in a purportedly dynamic formula. This latter form also came to be used with double-acting hammers when they were developed, although they are about as widely different from a single-acting hammer as the latter is from a drop hammer.

It has become customary to use the Engineering-News formula without consideration of the qualifications attached to it by its author in his original article.[5] Mr. Wellington accompanied the formula by the statement that it

[5] "Formulae for Safe Loads of Bearing Piles," by A. M. Wellington, *Engineering News*, December 29, 1888, pp. 511–512.

was intended

> "* * * for safe working loads for piles under all ordinary conditions, to be reduced under exceptional conditions (as notably with irregular penetration) but never exceeded unless the pile is known to rest on rock and act as a column * * * [it is] assumed to be sensible and at an approximately uniform rate [of driving]."

It seems to be common practice today to specify the average penetration under the last few blows. Reported values of *s* are not usually accompanied by any statement as to whether or not this figure represented fairly uniform driving over considerable distance, driving over the last few inches, or a sudden reduction in penetration per blow. In this paper it has been necessary to use the reported values, in line with common practice, without such distinctions as may have been intended by the author of the Engineering-News formula. However, this paper is intended to compare results obtained by everyday methods and practices.

The Hiley formula contains terms representing the temporary compression of the cap, pile, and ground, impact loss, efficiency, coefficient of restitution, and weight of pile—not appearing in the Engineering-News formula. The Hiley formula may be converted into the Engineering-News formula by disregarding the weight of the pile and by assuming the following: 100% efficiency; no impact loss; representation of half the sum of the temporary compressions of cap, pile, and ground by 1.0 in. if a drop hammer is used and 0.1 in. if a steam hammer is used, regardless of the length of the pile, its material, or whether the pile is friction or end bearing; and a safety factor of 6.

Any dynamic formula can give a true, permanent load-bearing value only if the soil resistance opposing the blow is the same as the resistance of the soil to continued uniform static load. In the case of cohesionless soils, such as sands and gravels, the force of the blow is resisted by the soil particles, because water in the voids is free to move elsewhere. Any formula of the dynamic type can be of value for load-carrying purposes only if the pile load is delivered into soils of this character. In clays the force of the blow is resisted by the water in the voids, because it cannot immediately flow elsewhere due to the relative impermeability of this type of soil. Water in the clay lubricates the sides of the pile during driving, since it cannot quickly escape. Friction between the pile and the clay will increase with time, and will approach or equal the shearing value of the undisturbed soil or the friction value between the undisturbed material and the pile face. However, general settlement under added load may take place at a slow rate, due to consolidation of the clay as the increased pressure on the pore water in the clay relieves itself by slow percolation of the entrained water.

Comparisons of values computed by the Hiley and other dynamic formulas with load test results have been published. These comparisons were intended to show that none of the formula results could be relied upon to agree with the actual ground resistance.[6,7] There appear, however, to have been two

[6] Discussion by Karl Terzaghi of "Pile-Driving Formulas: Progress Report of the Committee on Bearing Value of Pile Foundations," *Proceedings*, ASCE, February, 1942, pp. 311–323.

[7] Discussion by G. G. Greulich of "Pile-Driving Formulas: Progress Report of the Committee on Bearing Value of Pile Foundations," *ibid.*, September, 1941, pp. 1391–1396.

basic errors in application of the formulas—namely, use of dynamic formulas in cohesive soils and, in the case of the Hiley formula, consideration of the value of l as the full length of the pile regardless of the location of the center of driving resistance. Use of the incorrect value of l also results in use of the wrong average cross-sectional area of a tapered pile. Of the thirty tests tabulated and plotted, only three were in cohesionless soils. Good agreement between dynamic formula results and test loads could not be expected under these circumstances, where most of the soils were of a type in which no dynamic formula is applicable. However, the results for the three piles in cohesionless soils, as computed by an approximation of the Hiley formula, fell within practically the same satisfactory range as did a number of other tests in cohesionless soils reported herein. The data indicate that no formulas give results in good agreement with load tests when driving in cohesive soils, which finding agrees with the theories of soil mechanics. However, the results would not seem either to prove or to disprove the value of the Hiley formula, properly applied, under conditions where a dynamic formula is applicable.

Although dynamic formulas applied in cohesive soils will give the temporary resistance to the blow, care should be exercised to avoid considering this resistance as a permanent load-carrying capacity. By using a formula of the Hiley type under cohesive soil conditions, however, it is possible to compute the fiber stress in the pile during driving.[8]

Separation of Studies of Variables and Factor of Safety

A study of reliability of dynamic driving formula results as one measure for bearing capacity is best approached by separating it into two phases. In the first, it is well to ascertain whether or not a dynamic formula can be found which will give for an answer the same value of the ultimate driving resistance, R_u, for all types of piles driven with all types, sizes, and speeds of hammers to the same depth in the same soils. This reconciles all variables in pile characteristics and driving equipment. The second phase consists of studying the relationship between R_u and the ultimate bearing values as determined by test loads.

A separation of these phases enables the problem to be studied intelligently, and avoids confusing the effects of pile and hammer variables with the factor of safety. After a method of obtaining uniform values of R_u has been found, selection of the safe working value, R, by adoption of a proper factor of safety for the conditions of the particular structure is simple.

Reconciliation of Variables in Driving Conditions

Data on the first phase of study of the value of pile-driving formulas, reconciling driving results when using various types of piles and sizes of hammers, have been presented in tabular form elsewhere.[9,10] In these tests, the following combinations were investigated:

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[8] "Field Determination of Damaging Stresses During Driving," by Robert D. Chellis, *Engineering News-Record*, May 30, 1946, pp. 863–865.

[9] Discussion by Robert D. Chellis of "Pile-Driving Formulas: Progress Report of the Committee on Bearing Value of Pile Foundations," *Proceedings*, ASCE, October, 1941, pp. 1517–1537.

[10] "Pile-Driving Handbook," by Robert D. Chellis, Pitman Publishing Corp., New York, N. Y., 1944.

1. Different types of piles (thin mandrel-driven corrugated shells, fluted steel shells, and pre-cast concrete) were driven with the same hammer to the same depth.
2. Different types of piles (fluted steel shells, and thin mandrel-driven corrugated shells) were driven with the same hammer to the same capacity by the Engineering-News formula.
3. The same type of H-pile was driven with different types of hammers (double-acting and differential-acting).
4. The same type of pile (wood) was driven with different types of hammers (drop and double-acting).

These data appear to indicate that the Hiley formula will reconcile with variables of different hammers and pile types, and give closely the same computed driving resistance in any particular soil conditions at the same depth. This is as it should be, since the types of piles driven into it and subjected to load make no difference to the soil, provided the dimensions of the piles are comparable.

Comparison of Driving and Test Load Values

The second phase of study of the value of pile-driving formulas can now be investigated with more surety, on the basis that the pile and hammer variables can be reconciled within reasonable practical limits. This permits direct comparison of driving resistance values with the failure points of load tests. Basic data for forty-five test piles meeting resistance in cohesionless soils are given in Table 3. Also shown are the test values and ultimate driving resistances computed by dynamic formulas, as well as data on the hammers used in driving the piles. Data on the borings at the selected sites are given in Table 4.

The scattering of the ratios between the ultimate driving resistances computed by the various dynamic formulas in most common use and the failure points of test loads of the piles in Table 3 are shown in Fig. 1. Because the Engineering-News, Eytelwein, and Navy-McKay formulas are intended to give working loads with a safety factor of 6, the results computed by their use have been multipled by 6 to obtain ultimate resistances. Also, since the Canadian National Building Code formula contains a safety factor of 3, the results obtained thereby have been multiplied by 3 to secure ultimate resistances. The Hiley and Pacific Coast Uniform Building Code formulas give ultimate resistance directly.

The best and safest range of values in Fig. 1 appears to be obtained from the Hiley formula, falling between approximately 55% and 125%, with an average of 92%. No dangerously high percentages appear. The Pacific Coast formula shows a somewhat wider range—between approximately 55% and 220%, with an average of 112%. The Canadian formula range falls into somewhat lower figures, from 55% to 140%, with an average of 80%. The range of the Engineering-News formula is from 100% to 700%, with a 289% average, and that of the Eytelwein formula, from 90% to 1,800%, with an

average of 292%. The range of values by the modified Engineering-News formula is from 98% to 430% (182% average); that of the modified Eytelwein formula is from 98% to 508% (202% average); and that of the Navy-McKay formula is from 99% to ∞.

It may be observed in Fig. 1 that the results of the Hiley, Pacific Coast, and Canadian formulas are grouped in some proximity to the 100% test line, so that, with the factors of safety assumed, none would be actually unsafe, and none very wasteful. On the other hand, the scattering of results from the Engineering-News, Eytelwein, and Navy-McKay formulas is marked and too wide to be comprehended economically within any one factor of safety. Even a factor of 6 would not be adequate for some results.

TABLE 3.—Data on Piles, Driving, Load Tests,

Test No.	Location	Pile Characteristics: Type[a]	Length (ft)	Embedment (ft)	Butt size[b] (in.)	Tip size[b] (in.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(a) Timber Piles				
40	San Francisco Harbor, Calif.	Douglas fir, creosoted	92	73	16	8
30	Lavaca Bay Bridge, Port Lavaca, Tex.	Upper 32 ft creosoted (24 lb)	84	63	12[g]	9[h]
37	Marathon, Ont., Canada	Douglas fir, green	68.8	64.8	15	10½
44	Norfolk, Va., waterfront	Long leaf yellow pine	60	57.7	13½	8
45			70	67	17	7
33	Marathon, Ont., Canada	Douglas fir, green	63	59.5	14	7½
34			61	59.5	14	8
2	Pee Pee Creek Bridge, Pike County, Ohio	Wood, green	45	42	14	6
32	Marathon, Ont., Canada	Douglas fir, green	63	59.5	13½	8
31			75	71	13½	8
8	Kokasing River Bridge, Morrow County, Ohio	Oak, green	30	29	12	7
1	Crooked Creek Bridge, Pike County, Ohio	Wood	39	25	13	8
35	Marathon, Ont., Canada	Douglas fir, green	63	59.5	14	7½
		(b) Composite Pipe and Wood				
36	Marathon, Ont., Canada	Douglas fir[p]	40	25	12½	10
		(c) Fluted Steel Shell				
7	Scioto River Bridge, Pike County, Ohio	11-gage	25	25	18	8
6		11-gage, straight upper section	45	26	18	8
4	Pee Pee Creek Bridge, Pike County, Ohio	11-gage	25	19	18	8
		(d) Pipe Piles				
3	Brush Creek Bridge, Adams County, Ohio	12¾ in. in diameter, ⅜-in. wall	50	47	12¾	12¾
43	Cuyahoga River Turning Basin, Cleveland, Ohio	12¾ in. in diameter, ½-in. wall	80	78	12¾	12¾
42			80	70	12¾	12¾

Curves showing the relationship between test loads and driving resistances computed from the Hiley and the Engineering-News formulas—both placed on a working load basis by the application of factors of safety—are given in Fig. 2. The Engineering-News formula purports to include a factor of safety so that results are plotted directly. A safety factor of $2\frac{1}{2}$ has been assumed sufficient for use with test loads and the Hiley formula. The values in Fig. 2 have been grouped according to the type of pile and have been placed in descending order of set values within each group. The increasing divergence of the Engineering-News values as sets became smaller should be noted. There are many other variables involved, but this trend is apparent. The divergences appear to start at larger sets with the heavier piles. Curves of the Pacific

TESTS, AND ULTIMATE CAPACITY BY FORMULAS

Set, s (in.)	HAMMER: Type	HAMMER: Energy per blow[c]	HAMMER: Blows per minute	Pile cap weight (lb)	Pile weight,[d] W_p (lb)	Failure test load (tons)	ULTIMATE CAPACITY BY FORMULA (TONS): Hiley	Pacific[e]	Canadian[f]	Engineering-News[g]	Eytelwein[e]	Modified Engineering-News[g]	Modified Eytelwein[e]	Navy-McKay[e]
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
PILES														
3.0	Drop	2,900 ×240	....	....	3,700	45	45	47	45	88	52	....	....	....
2.45	Drop	3,290 ×60[i]	....	1,080[j]	3,130	28	17	24.5	21.3	28.5	24.6	....	....	....
2.10	S.A.[k]	5,000 ×36	....	....	2,750	25	18	23	22	40	42	38	39	35
1.0	S.A.	5,000 ×36	....	65	3,025	44[l]	37.5	47	49	81	85	69	76	76
0.86	S.A.	5,000 ×36	....	65	3,115	40[m]	43.5	52.5	57	94	98	77	85	88
0.6	D.A.[n]	6,790	128[o]	....	2,200	40	23	22.5	22	58	54	45	39	46
0.54	D.A.	7,240	132[o]	....	2,200	40	23	26	25.5	68	63	53	45	56
0.48	S.A.	5,000 ×35[o]	....	100	1,400	71	68	71	75	150	148	134	188	204
0.46	D.A.	7,240	132[o]	....	2,200	40	28	27	27	78	73	57	47	65
0.4	D.A.	8,200	140[o]	....	1,820	43	34.5	32	34.5	98	95	70	65	90
0.33	S.A.	3,000 ×25[o]	....	40	780	37	45	48	49	87	105	60	92	104
0.3	S.A.	3,000 ×29[o]	....	100	1,300	45	50	51	53	109	144	72	102	129
0.19	D.A.	8,200	140[o]	....	2,100	40	44.5	39	43.5	170	147	100	76	176
WOOD PILES														
2.00	S.A.	7,500 ×38[o]	....	....	1,800	26	32	45.5	32	75	78	68	75	68
SHELL PILES														
0.43	S.A.	5,000 ×35[o]	....	670	1,100	88	94	100	50	166	206	108	192	198
0.5	S.A.	5,000 ×35[o]	....	670	1,500	78	72	76	46	146	174	108	174	168
0.50	S.A.	5,000 ×35[o]	....	670	1,100	88	83	93	48	146	177	108	168	170
PILES														
0.12	S.A.	3,000 ×29[o]	....	600	3,100	46	38	79	53	194	164	102	115	276
0.10	S.A.	5,000 ×36[o]	....	1,080	3,250	120[q]	85	107	74	450	626	225	390	798
0.09	S.A.	5,000 ×36[o]	....	690	3,250	150[r]	86	108	74	474	556	230	300	822

TABLE 3.–

Test No.	Location	Pile Characteristics: Type[a]	Length (ft)	Embedment (ft)	Butt size[b] (in.)	Tip size (in.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
						(e) Ste
16	Wharf, Bremen, Germany	10-in., 60 lb per ft	39.3	35.4	10	10
19			39.3			
11			39.3			
12			39.3			
13			39.3			
15			39.3			
17			39.3			
21			29.3	25.4		
22			29.3	25.4		
20			39.3	35.4		
26	Bay Bridge Falsework, San Francisco, Calif.	12 in., 65 lb per ft	99.2	43.2	12	12
14	Wharf, Bremen, Germany	10 in., 60 lb per ft[e]	27	23	10	10
18		10 in., 60 lb per ft	39.3	35.4	10	10
39	Cowen Park Bridge, Seattle, Wash.	10 in., 42 lb per ft	46	32	10	10
41	Cuyahoga River Turning Basin, Cleveland, Ohio	10 in., 54 lb per ft	80	78	10	10
24	Bay Bridge Falsework, San Francisco, Calif.	12 in., 65 lb per ft	98.8	40.5	12	12
27			99.5	44.7	12	12
10	Alameda Creek Bridge, Niles, Calif.	10 in., 42 lb per ft	38	35	10	10
25	Bay Bridge Falsework, San Francisco, Calif.	12 in., 65 lb per ft	99	44.4	12	12
28			99.6	40.4	12	12
29			100.1	41.4	12	12
9	Strip Mill Foundations, Lackawanna, N. Y.	10 in., 57 lb per ft	30.8	24.7	10	10
23			40	30	10	10
						(f) Pre-ca
5	Scioto River Bridge, Pickaway County, Ohio	12 in. by 17 in.	14	11	12×17	12×1
38	Naval Base, San Diego, Calif.	20 in. by 20 in.	53	....	20	20

[a] Wood piles untreated unless specifically noted. [b] For round piles, diameter; for square piles, one side; f
[d] Includes cap weight. [e] Pacific Coast Uniform Code. [f] Working load × 3. [g] Working load × 6. [h] Upp
[j] Wood dolly in steel cap, rope coil on pile. [k] Single acting. [l] Day of driving. [m] 30-day duration. [n] Doub
in diameter by ⅜ in. thick by 50 ft long. [q] Compression test 7 days after driving; 140 tons pulling test 17 da
sets of projecting fins tilted from horizontal. [r] Compression test 9 days after driving; 85 tons pulling test

Coast and Canadian formulas have not been shown in Fig. 2 since they would be confusing and since the values group well with the Hiley values, as may be seen in Fig. 1. The similarity in ranges between the Eytelwein, Engineering-News, and Navy-McKay formulas, including the modified forms, have likewise resulted in the omission of curves for these formulas.

The reasonable coincidence of the curves in Fig. 2 for wood piles, for the easier drivings, indicates that the actual factor of safety provided by the Engineering-News formula is often nearer $2\frac{1}{2}$ than 6. There would usually be no objection to this, and such a result would be desirable, but reliance should not be placed on the existence of a safety factor having a value of six times the working load. Fluted steel shell piles are lightweight, and appear to have

(Continued)

Set, s, (in.)	Hammer: Type	Hammer: Energy per blow[c]	Hammer: Blows per minute	Pile cap weight (lb)	Pile weight,[d] W_p (lb)	Failure test load (tons)	Ultimate Capacity by Formula (Tons): Hiley	Pacific[e]	Canadian[f]	Engineering-News[g]	Eytelwein[g]	Modified Engineering-News[g]	Modified Eytelwein[g]	Navy-McKay[g]
(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
(e) Steel H-Piles														
0.76	....	10,850	....	....	....	56	42.5	48	38	76	78	61	64	70
0.76						44.5	42.5	48	38	76	78	61	65	70
0.66						45	50	54	40	86	88	68	72	80
0.63						55	55	57	42	90	92	70	75	84
0.55						67.5	54	61	52	100	104	77	82	96
0.47						45	60	71	50	114	118	84	92	113
0.47						79	62	71	50	114	118	84	93	113
0.45						45	61	71	50	114	118	84	93	113
0.45						67	61	71	50	114	118	84	93	113
0.36						89	72.5	84	48	141	154	99	109	144
0.36	D.A.	18,920	110[o]	....	7,200	88	65	78	57	244	210	171	126	204
0.30	....	10,850	....	....	....	78.8	80	100	57	162	171	109	120	177
0.30	....	10,850	....	....	....	112+	80	100	57	162	171	109	120	177
0.27	S.A.	5,000×36	....	750	2,680	70+	92	152	84	243	291	300	780	∞
0.17	S.A.	5,000×36[s]	....	660	4,900	75[r]	79	100	66	336	353	192	210	426
0.14	D.A.	18,920	110[o]	....	7,200	90	83	100	67	472	360	258	168	534
0.13	D.A.	18,920	110[o]	....	7,200	80	84	104	70	492	372	264	172	564
0.12	S.A.	3,000×29	....	....	2,300	99	75	72	60	194	244	102	123	300
0.08	D.A.	18,920	110[o]	....	7,200	140	89	108	69	636	444	258	170	528
0.06	D.A.	18,920	110[o]	....	7,200	150	92	110	72	729	496	318	194	1,332
0.05	D.A.	18,920	110[o]	....	7,200	140	93	114	75	755	504	324	195	1,494
0	S.A.	5,000×36	....	....	2,500	190	175	210	144	875	1,800	300	360	∞
0	S.A.	5,000×36	....	....	3,000	128	157	248	90	900	1,920	300	654	∞
(f) Pre-cast Concrete Piles														
0.26	S.A.	5,000×28[o]	....	1,480	4,460	70	75	153	42	192	218	124	158	225
0	S.A.	7,500×34[o]	....	....	22,550	150+	180	214	82	1,380	523	520	174	∞

rectangular piles, both dimensions. [c] Ram weight (lb) times stroke (in.); or manufacturer's rated energy, in ft-lb. sections only; lower section 9 in. to 13 in. butt diameters, 8 in. to 9 in. tip diameters. [i] Cable drag on drum. acting. [o] Measured. [p] Driven through casing extending to lower sand stratum, using pipe mandrel 10¾ in. after driving. [r] Compression test 10 days after driving; 80 tons pulling test 16 days after driving. [s] With 4 days after driving.

many of the driving formula characteristics of wood piles. A similarity between the curves is, therefore, to be expected, as may be observed in Fig. 2.

As piles become heavier, the Engineering-News, Eytelwein, and Navy-McKay formulas, including the modified forms, become more dangerous to use with small sets. For the tests plotted, it would not be safe to use them with sets of less than ¼ in., although there are so many possible variables involved that this should not be considered a rule.

The lighter the hammer, the smaller the set will be and, consequently, the larger the computed carrying capacity by the Engineering-News formula. According to common specifications in which no limit is set on the weight of the pile or the smallness of the hammer, this formula could result, when using a

very small hammer, in tiny sets or refusal at short penetrations. Thus, it is seen that the length of pile driven would depend on the size of hammer or weight of pile, instead of on reaching some definite stratum capable of supporting the load safely with a proper factor of safety. At refusal the Navy-McKay formula gives infinity as the ultimate bearing, which makes the results doubtful when sets are so small as to approach refusal.

TABLE 4.—Soil Conditions for Pile Tests Listed in Table 3

Test No.	Soil Conditions
1, 2, 5, and 6	Sand and gravel
3	14 ft silt, clay, and fine sand; 10 ft clay; gravel, fine sand, and broken stone
4	12 ft silt; sand and small gravel
7	25 ft silt, sand, and gravel; sand and gravel
8	12 ft silt, fine sand, and gravel; sand and gravel
9	6 ft clay; 2 ft sand and gravel; 13 ft soft clay; 4.7 ft sand and gravel to rock
10	Gravel and small boulders
11	28 ft sand; 0.5 ft hard clay; 2 ft clay with sand; 5 ft fine sand
12, 13	23–24 ft sand; 1.5–2 ft clay; 0.5 ft peat; 0.5–1 ft clay; 8–10 ft sand
14, 39	Fine sand
15, 16	13 ft sand; 1–2 ft loam; 10 ft fine sand; 1–1.5 ft clay; 3–6 ft clay and sand; 3–7.5 ft fine sand
17, 18	23–25 ft sand; 2 ft clay; 0.5 ft peat; 1 ft clay; 7–8 ft clayey sand
19	22.5 ft sand; 1.5 ft clay; 1 ft sand and clay; 9.5 ft sand
20	23 ft sand; 1 ft clay; 1 ft sand; 3 ft sandy clay; 6.5 ft sand
21	23 ft fine sand; 1 ft clay; 1 ft peat; 0.5 ft sandy clay
22	18 ft fine sand; 4 ft sand and silt; 2 ft clay; 1.5 ft sand
23	7.5 ft clay; 3 ft sand; 10.5 ft soft clay; 6 ft sand and gravel to rock
24 to 29	40 ft water; 13 ft soft mud; 20 ft clayey sand; 18 ft sandy silty clay; 37 ft silty clay
30	Silt with some shell strata
31	10 ft sand; 5 ft silt; 2 ft sand; 38.5 ft plastic clay; 8 ft silty sand; 8 ft fine sand[a]
32 to 35	5 ft sand; 39.5 ft plastic clay; 3 ft plastic silt; 12 ft fine sand[a]
36	14 ft sand; 32 ft plastic clay; 60 ft fine sand[a]
37	5.5 ft sand; 37 ft plastic clay; 44 ft silty sand[a]
38	Water, mud, and sandy clay to firm sand
40	Harbor mud
41, 42, 43	7.5 ft sand and loam; 26.5 ft fine sand; 1 ft gravel; 59 ft silty sand and stiff dry clay
44, 45	9 ft sandy silt; 12 ft soft silt; 6 ft sand and silt; 18 ft soft clay, some sand; 4 ft silt, some peat; 4 ft silty sand; 75 ft fine sand, shells, and some clay

[a] Under hydrostatic pressure.

The reason for the danger in the use of the Engineering-News and the Eytelwein formulas with small sets may be understood from consideration of the relative values of the terms in the denominators. The last term of 1.0 for drop hammers or 0.1 for other hammers in the Engineering-News formula, and the last term of 0.1 W_p/W_r for single-acting or double-acting hammers with the Eytelwein formula, are intended to represent the average elastic compressions. If the average elastic compressions, which are half of those observed at the head, are reasonably close to these figures, the results might not vary too widely. Nevertheless, under hard driving the elastic compressions may actually be many times these values and may exert a far greater reduction on the result than the set. As sets decrease, elastic compressions always increase. The relationship between set and compression would be different for each type of pile. At any particular driving resistance there will occur specific values of set and compression, but because these values are always changing in inverse ratio no such formula (the Engineering-News or the Eytelwein), which has a fixed value in the denominator, could be expected to apply to all conditions. Sets and elastic compressions are equally important in a formula, and it is no

more logical to assume a constant value for the temporary compression than it is to assume it for the set. The Engineering-News formula was intended for use with wood piles, but is not suited to hard driven piles, or to heavy piles.

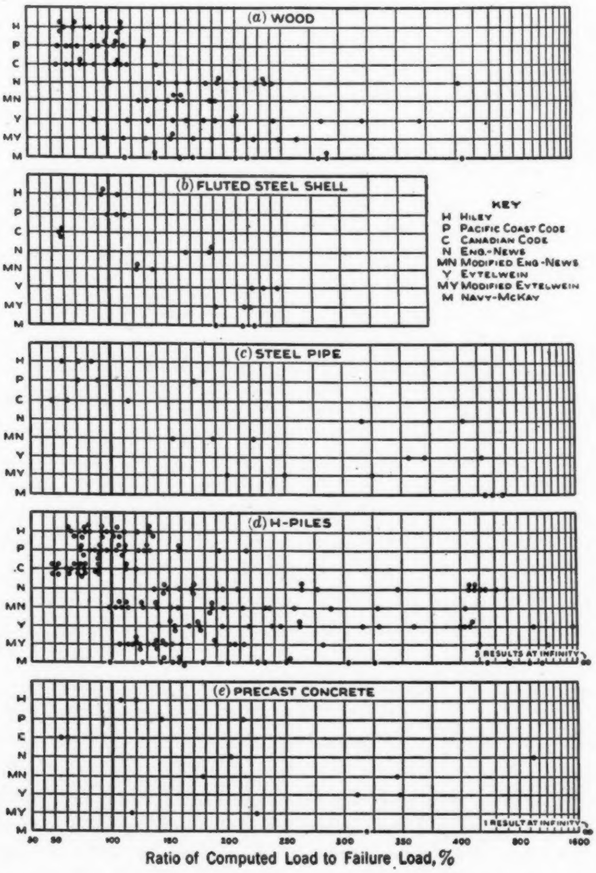

Fig. 1.—Scattering of Ratios Between Resistances Computed by Various Pile Formulas and Observed Failure Test Loads

Failure Point in Load Tests

The point at which failure is considered to occur in a load test provides the basic value against which the pile-driving formula results are compared. Load is always plotted against settlement in load test curves. Settlement of the head is composed of elastic shortening of the pile and ground, and of movement of the pile relative to the ground or plastic deformation of the ground. Rebounds measured after temporarily removing the load before reapplying it with the next load increment will enable these two forms of settlement to be separated for consideration. The graph of total head movement can then be

plotted as two separate elastic and plastic settlement curves.[11] It is only permanent movement which actually governs the failure point.

If the theoretical elastic pile deformation under the loads, computed on a length from butt to estimated center of soil resistance, is plotted on the load settlement curve, the curve of elastic deformation of the pile may be compared with it.[11] Until the elastic deformation curve drops as low as the theoretical elastic line, the center of resistance is higher than assumed, and the upper strata are being tested instead of the lower strata.

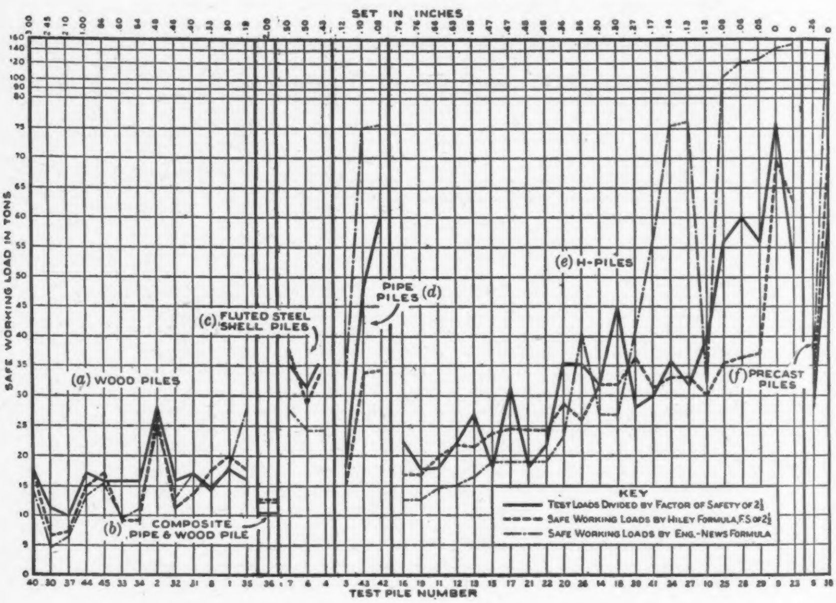

FIG. 2.—RELATION BETWEEN PILE FORMULA AND TEST LOAD RESULTS

Certain strata are often selected as providing the bearing resistance, and the upper part of the pile is intended to by-pass the overlying strata. In such cases, the value of the temporary load-carrying capacity of these upper strata should be subtracted. The load test should be carried to such a point that a satisfactory amount of load reaches the lower strata to be tested. Many load tests are not of sufficient magnitude relative to the working load. Building codes often require test loads to be 1.5 or 2.0 times the working load. It would be advisable in many instances for the engineers to carry the test to a greater value, and preferably to failure so that the factor of safety may be known.

After its separation from the total head settlement curve, the curve of permanent movement should be inspected for the failure point. Failure is usually considered to occur where the rate of movement begins to increase

[11] "Making and Interpreting Pile Load Tests," by Robert D. Chellis, *Engineering News-Record*, June 13, 1946, pp. 914–919.

sharply in proportion to the increase in load. It is an obvious point in some tests, whereas in others the curve appears to suffer a gradual increase in the rate of change of slope, thereby making selection of the failure point difficult.

Judgment and experience have resulted in many rules to aid in selecting the failure point. Some of these are contained in building codes, some in textbooks and articles, and others are matters of practice among various engineers. A partial summary has been published.[11] Sometimes when difficulty is experienced in applying certain rules, replotting the curve to a different scale of settlements, or plotting it on semilogarithmic paper, will bring out the points more clearly. On a recent test, application of fourteen of these rules gave working loads ranging from 11 to 18 tons, with an average of $14\frac{1}{2}$ tons.[11] If difficulty is experienced in applying one of the rules, several or all these and others may be applied and the results averaged. The discrepancy between the result and the true value would not seem likely to be larger than could be covered by the usual factors of safety.

A load test may reveal that there has been an increase in bearing value of the soil over that computed, and, on the other hand, it may reveal a decrease. Cases have been reported where coarse-grained, saturated pervious soils have lost up to 40% of the indicated bearing value in 24 hours after driving.[12]

Very loose fine-grained sands and silts are occasionally encountered, and if these are submerged and below critical density, excess water temporarily forced away from the pile during driving may cause a momentary "quick" condition, resulting in low resistance readings, although static load tests would show much higher values.[13]

Factor of Safety

It has often been said that the pile foundations have generally been satisfactory, and that this justifies continued reliance on the Engineering-News formula. Most foundations have not failed. Only those would fail, presumably, in which the actual factor of safety fell below 1.0.

In this connection, it should be borne in mind that full design live load is generally not applied. Careful studies have shown that only a small percentage of the usual office building live loads actually occurs. The same would seem to be true of some other structures. Furthermore, some live loads are transient, whereas others are of insufficient duration to cause squeezing out of the water and settlement in upper cohesive strata, which thus reduce the load on those sections of piles in the strata capable of providing permanent resistance.

There is, however, an impressive list of failures, even though these are not generally publicized; and many instances occur in which the factor of safety is very low—far below that of the balance of the structure. Excess footage of piles frequently has been driven. Piles have been overdriven and broken, as found when they have been pulled or excavated (Fig. 3). When using piles of different types, such as light shells or pre-cast concrete, widely different tip grades have resulted, one of which must surely be wrong. The same

[12] "Soil Reactions in Relation to Foundations on Piles," by R. M. Miller, *Transactions*, ASCE, Vol. 103, 1938, pp. 1193–1216.

[13] "Application of Soil Mechanics in Designing Building Foundations," by A. Casagrande and R. E. Fadum, *ibid.*, Vol. 109, 1944, p. 383.

result occurs when using different types or sizes of hammers on piles of the same kind.

Fig. 1 reveals that the Engineering-News, Eytelwein, or Navy-McKay formulas, or their modified forms, cannot be expected reliably to keep the results within a spread which can be encompassed by any reasonable factor of safety. Even the purported value of 6 used is not adequate for this purpose. The comprehensive formulas fall within a reasonably narrow spread, the Hiley formula giving the smallest scattering of values.

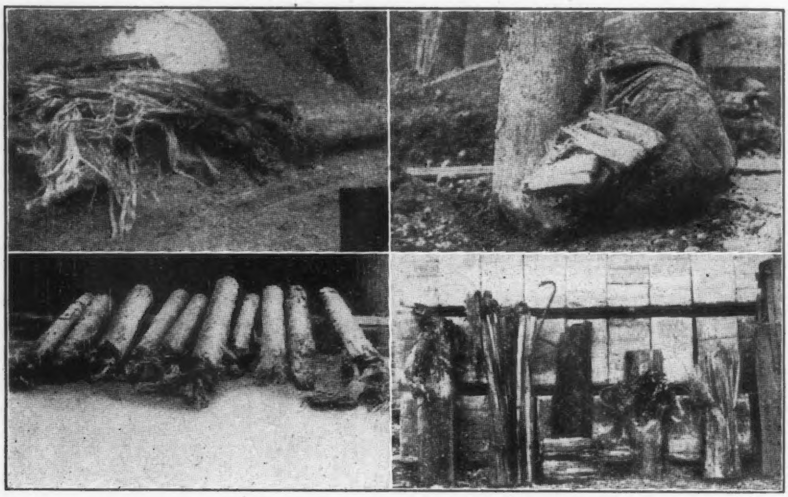

FIG. 3.—EXAMPLES OF OVERDRIVEN AND BROKEN PILES

Comparison of results obtained by the Engineering-News formula with those obtained by the Hiley formula have shown a relative variation in the factor of safety from as low as $\frac{1}{2}$ to as high as 16. In this particular series of tests comparison of results obtained by the Engineering-News formula with load test results showed an actual range in the safety factor of from $\frac{2}{3}$ to 6.

The spread of computed results by the Hiley formula, compared with load tests, is such that it can be used with a safety factor of $2\frac{1}{2}$ or 3. This compares favorably with the safety factors used in structural steel and concrete designs. The uncertainties in interpretation of failure values from load test curves, and of selection of hammer efficiency, center of driving resistance, and cap and ground quake values in the pile formula seem to be hardly more broad than the uncertainties and neglect of conditions of end restraint, for instance, or the theory of concrete design, used in the superstructure. The safety factors used in superstructure designs are intended to cover differences between theory and actual conditions, as well as provide a true "factor of safety." For this reason, it would be unfair to expect a higher degree of accuracy in the design procedures for substructure, and not to permit the so-called factor of safety some latitude in covering discrepancies between theory and actuality.

Reasonable reconciliation of driving results by the Hiley formula with load test results permits the use of any selected factor of safety. This may be chosen with regard to the economic importance of the structure, its expected life, the reliability of data based on completeness of boring, driving and test information, the uniformity of soil conditions, and other variables.

Static Formula

It is recommended as good practice to use a static formula in preliminary considerations of pile problems and selection of test pile lengths. It is also advisable to use a static formula to check values computed by a driving formula, particularly if a load test is not made.

The static formula recommended merely consists of multiplying the bounding area of the pile by the square-foot friction values estimated for the various soil strata, plus an amount for end bearing. The friction values may be judged from boring records, shear tests, loading and pulling tests, and experience. A table of such values for various types of soils, determined from load tests, has been published.[10] This is a rough guide as to the possible range when more accurate data are lacking.

Limitations Affecting Value of Piles

In considering the value of either a computed resistance or a load test on a single pile, two points should be borne in mind. First, the load-carrying capacity of a group of piles is often less than that given by multiplying the number of piles in the group by the value of a pile tested or driven singly, due to overlapping of zones of influence. Second, no pile can improve poor strata below its tip, and if the character of the underlying soils is such that they will consolidate under the load of the structure, this will not be prevented by driving piles. Reduction due to group action is not a fully understood subject. It is likely that it affects piles driven in cohesive soils much more than in cohesionless ones,[10,14,15] but it may be of considerable magnitude. For the theory of consolidation of cohesive strata any textbook on soil mechanics may be consulted.

Conclusions

It would appear that the Hiley, Canadian National Building Code, and Pacific Coast Uniform Building Code formulas are practical tools for use in pile foundation design (within the limits of applicability of individual pile values and underlying soil conditions) in soils in which the predominant driving resistance is supplied by cohesionless materials. It further appears that these formulas would not result in uneconomical foundations, and that they would provide ultimate driving resistance values in sufficiently good agreement with test values to permit their use with factors of safety comparable to those in use for other phases of structural engineering. These formulas are also useful in cohesive strata for determining the temporary resistance of the pile under the blow, as a means of computing the fiber stress during driving.

[14] "Timber Friction Pile Foundations," by F. M. Masters, *Transactions*, ASCE, Vol. 108, 1943, pp. 115–173.

[15] "The Efficiency of Piles in Groups," by J. F. Seiler and W. D. Keeney, *Wood Preserving News*, November, 1944, pp. 109–118.

For each pile the length and weight used in the Hiley formula were the effective length to the center of driving resistance and the total weight of the pile plus driving cap. In the results obtained by the Canadian and the Pacific Coast formulas, the pile lengths were taken as full lengths, and pile weights did not include cap weights, to agree with these terms as defined by the authorities sponsoring the two formulas. Had these terms been applied as recommended and used for the Hiley formula, some variations would have resulted, but the scattering would not have been as small as obtained by that method.

Consideration of the assumptions made in the Engineering-News formula makes it evident that the formula cannot be of universal value in these days of such wide ranges of piles and equipment, even though there are certain combinations of values in which it will give good results. It would seem from the Mr. Wellman's original qualifications that this formula was intended for friction piles rather than for piles which resist driving somewhat suddenly and become, in effect, end-bearing piles or columns. Many of the unduly high results from this formula would be avoided if such cases of fairly sudden small sets were eliminated. However, to be sure that it is within the safe range, it is necessary to run a loading test or compare with results by the Hiley formula, unless full reliance is placed upon a static formula. There would not seem to be much point in continued use of the Engineering-News formula, except as a matter of interest in comparing it to results of more modern methods and becoming aware by personal experience of its limitations. Many building codes still retain it, but the engineer should not rely upon it in such cases, and for reasons of safety, uniformity, and economy should apply more accurate methods as well. Most code authorities will accept load test results modified by a reasonable factor of safety, if code formulas appear to cause uneconomical pile lengths.

Although use of the modified forms of the Engineering-News and Eytelwein formulas with heavy piles appears to reduce the range of results compared to load tests, these tests do not indicate that sufficient improvement is obtained to justify their use instead of a formula of the Hiley type. Neither do these tests indicate justification for use of the Navy-McKay formula.

On many small jobs load tests are not practicable. On most projects it is desired to obtain an idea of the carrying capacity or length of piles required before the time necessary to make an adequate load test elapses. If the borings show soil of a nature in which a dynamic driving formula is usable, it appears that sufficiently reliable results for these purposes may be computed. On large projects these values should be confirmed by a load test.

ACKNOWLEDGMENTS

Thanks are due to W. H. Rabe of the Bureau of Bridges, State of Ohio, for the data on tests 1 to 8, inclusive; to E. M. Cummings, M. ASCE, of the Bethlehem Steel Company, Bethlehem, Pa., for the data on tests 9 to 29, inclusive, and on test 39; and to G. B. Sowers, M. ASCE, Director of Public Works of the State of Ohio, Columbus, Ohio, for the data on tests 41, 42, and 43. Some of the data for tests 11 to 22, inclusive, were published by A. Agatz.[16]

[16] "Der Rammstahlpfahl fuer Pfahlvostbauwerke," by A. Agatz, *Die Bautecknik*, February 2, 1934, pp. 56–60 and February 9, 1934, pp. 68–71.

Some of the photographs of damage to overdriven piles were provided by the courtesy of the Raymond Concrete Pile Company, New York, N. Y. The data for tests 30, 38, and 40 are available elsewhere.[17,18,19]

APPENDIX. NOTATION

The following letter symbols have been adopted for use in this paper:

A = average of cross-sectional area of driven parts of pile at butt and center of resistance to driving, in square inches;

C = temporary compression allowance, in inches (when without subscript is defined by Eq. 10*d*):

  C_1 = for pile head and cap;

  C_2 = of pile;

  C_3 = of ground (quake);

E = modulus of elasticity of pile material;

E_n = rated energy of blow, in foot-pounds, as published by manufacturers for various speeds of hammer;

e = coefficient of restitution: 0.5 for ram striking steel anvil on steel or pre-cast concrete piles; 0.4 for ram striking steel anvil on wood piles, also for steel helmet containing wood and driving steel piles; 0.32 for ram striking on steel plate cover of wood cap of steel piles; 0.25 for ram striking well-conditioned wood caps on pre-cast concrete piles, also driving directly on wood piles; and 0 for deteriorated condition of heads of wood piles or of wood caps, and for excess packing in cap;

e_f = efficiency: Suggested as 100% for trigger released drop hammers; 75% for friction winch drop hammers, differential-acting steam hammers, and single-acting steam hammers; and 85% for double-acting steam hammers—frequently from 5% to 10% greater values (or values under 100%) are obtained under favorable conditions, sometimes less under unfavorable; use of listed values suggested for computing bearing values, and of higher values for computing stresses in piles;

H = height of free fall of ram, in feet;

h = height of free fall of ram, in inches;

L = length of pile, in feet, measured from head to center of resistance to driving (defined as full length of pile in Pacific Coast Uniform Building Code);

l = length of pile, in inches, measured from head to center of resistance to driving (defined as full length of pile in Canadian National Building Code);

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17 "Pile Tests Justify Increased Loading," by W. A. King, *Engineering News-Record*, June 28, 1945, p. 73.

18 "Driving and Loading of Concrete Test Piles at the Naval Supply Depot, Naval Operating Base, San Diego, Calif.," *Bulletin No. 36*, Public Works of the Navy, Washington, D. C., October, 1927.

19 Correspondence on Reinforced-Concrete Piles Driven in a Gravel Fill at Vancouver, B. C., by H. E. Squire, *Minutes of Proceedings*, Inst. C. E., Vol. 226, 1929, pp. 103, 108, and 109.

R = safe carrying capacity of pile, after applying factor of safety, in pounds;

R_u = ultimate carrying capacity of pile (considered as ultimate resistance to driving, in pounds, before applying factor of safety);

s = final set per blow, in inches;

W_p = weight of pile, in pounds, including shoes and driving cap and also of follower, or driving cores and mandrels, if used; shoes and caps not specified to be included in weight by Pacific Coast and Canadian building codes; and

W_r = weight of falling mass, in pounds.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

STABILITY OF THIN-WALLED COMPRESSION MEMBERS

By H. Dudley Wimer, Jr.[1]

Synopsis

Stainless steel has come into the foreground as a material for structures in which lightweight is of primary importance. Its weight economy is inherent in its high strength, permitting the use of thinner sections. Ordinarily, the form in which it is used is that of members fabricated from cold rolled sheet. The thinness of the sections used and the manner of their fabrication tend to make local instability a major factor in their design. This paper describes a method of computing the local stability, or crippling strength, of thin-walled compression members which was developed empirically from test data. Although derived primarily for stainless steel, the method is readily adaptable to other metals. Also, although the tests were conducted on small sections characteristic of aircraft structures, the principles discussed are equally applicable to large-scale members common to civil engineering practice.

Introduction

It is well known that long, slender compression members may fail by buckling, or bending sidewise, at a stress lower than the ultimate fiber compressive stress. This is known as column failure, or primary instability. Hollow, thin-walled members may exhibit a second form of instability known as local instability, or crippling. In this type of failure, the thin walls crumple. The stress at which this occurs depends on the shape and wall thickness of the member.

Although the study of local instability is not new and although there is a large volume of existing dissertation on the subject, the theory is still difficult to apply. Complications are introduced by moduli of elasticity which vary with the stress, lack of clear cut yield-point stresses, and unknown edge restraints and boundary conditions. High-efficiency sheet metal structures,

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1948.

[1] Chf. Analyst, Eng. Development Div., Rheem Mfg. Co., Lester, Pa.

such as aircraft and all other structures in which guaranteed strength values must be predicted closely, demand many tedious calculations to establish their compressive strength. Even then, the results are so unreliable that they must be substantiated by many physical tests.

This paper describes a method of computing the local stability or crippling strength of thin-walled compression members, which was developed empirically from test data, has been thoroughly proved in practice, and permits great rapidity and ease of calculations. This method was derived for use with 18-8 stainless steel of various hardnesses, having ultimate tensile strengths of from 75 kips per sq in. to 185 kips per sq in. The term "18-8" is a common trade designation[2] for a high-strength chromium nickel alloy. However, it is readily adaptable to other materials having stress-strain curves of the same general shape, and the same approach may be applied to a great variety of structural materials. Test specimens were made with their longitudinal axes parallel to the direction of cold rolling. This is in agreement with usual shop practice and is conservative. The compressive strength is considerably greater across the grain than parallel to the grain.

In comparing stainless steels of different hardnesses, the ultimate tensile strength rather than the yield strength is used as an index. This is done because 18-8 stainless steel has no clearly defined yield point. Tests have indicated that an arbitrarily defined yield strength has little physical significance with respect to local instability. A series of tests, specifically directed toward determining the influence of yield strength on the crippling strength of stainless steel, was run independently of the tests described herein. The specimens (see Fig. 1) were all square tubes with walls 0.050 in. thick. They consisted of three groups, geometrically identical, differing only in the physical properties of the materials from which they were made.

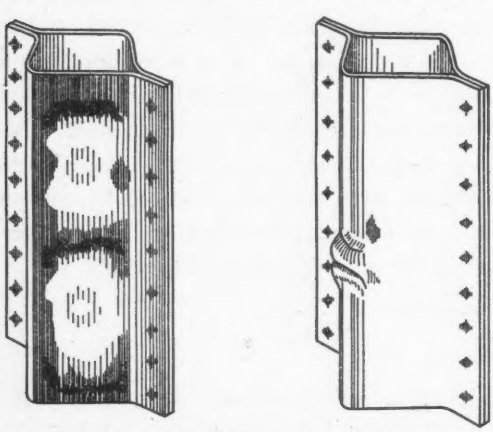

FIG. 1.—TYPICAL SQUARE TUBE SPECIMEN

Group I comprised five specimens, $3\frac{1}{16}$ in. long, with sides ranging in width from 0.55 in. to 2.55 in. This group was made from stainless steel having a yield strength of 129 kips per sq in. and an ultimate tensile strength of 156.5 kips per sq in. Group II was dimensionally identical to Group I but made of steel with a yield strength of 104 kips per sq in. and an ultimate tensile strength (UTS) of 154 kips per sq in. Group III was of steel with yield strength s_y of 81.4 kips per sq in. and UTS of 155.9 kips per sq in.

[2] *Transactions*, ASCE, Vol. 102, 1937, p. 1258, Table 4, item Nos. 5 to 10.

The three groups represented wide differences in yield strength with very nearly the same ultimate tensile strength. Comparable specimens in all groups failed at essentially the same compression load, the greatest single discrepancy being about 10%, based on the average.

Some introductory remarks are necessary to justify the practice, in this paper, of ignoring first buckling as the limit of useful strength. This is a familiar idea to aircraft structural designers, but will not be readily accepted by civil engineers. It is not intended to suggest that structures should be so designed that, under day-to-day service loads, buckling will occur in any of the plates. Also, it is realized that, with structures of the usual proportions found in civil engineering practice, there is probably a rather small margin between the stress at which first buckles appear and that at which permanent damage or failure occurs. However, a knowledge of the ultimate strength becomes very useful under certain special conditions:

(1) When plates of greater width-to-thickness ratios are used (as may be encountered with the use of high-tensile steels), the margin between first buckling and failure becomes greater. This is equally applicable to plates 4 ft wide and 1 in. thick and to sheets 2 in. wide and 0.04 in. thick. Improved data on buckling and behavior after buckling may enable the civil engineer to work closer to such stresses than he normally does at present. There is economic justification for close design in that the huge scale of many civil engineering projects may result in the wastage of tons of material, if ultraconservative methods of design are adopted.

(2) Under certain extraordinary loading conditions, buckling may be permissible as long as strength specifications are met. For example, in railway passenger car design, buckling is not desirable under normal operating loads but is acceptable under loads set up for collision conditions. In civil engineering, analogous conditions may arise from hurricanes, earthquakes, or other unusual sources of loading.

(3) Some structures are designed to fail at a predetermined load in order to safeguard other property. An example is a flashboard on the spillway of a dam, which is designed to collapse before the water reaches a sufficient head to endanger the dam itself. In cases of this nature, the prediction of ultimate strengths within close limits is essential.

Notation

The letter symbols used in this paper are defined where they first appear, in the text or by diagram, and are assembled alphabetically in the Appendix.

Crippling of Flat and Curved Elements Under Axial Compression Loads

Before presenting the test results, some review of the theory and procedure followed will be helpful. The basic idea has been used for some time by aircraft designers.[3,4] Structural members made from sheet metal, such as those

[3] "Airplane Structural Analysis and Design," by Ernest E. Sechler and Louis G. Dunn, John Wiley & Sons, Inc., New York, N. Y., 1942.

[4] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1943.

used in aircraft construction, are usually of such a nature that they may be broken down into flat and curved elements, the curved elements being the corners or bends (see Fig. 2).

The curved elements, or bends, may be thought of as segments of a hollow cylinder, whereas the "flats" are analogous to plates with a simple support along two opposite edges. The problem resolves itself, therefore, into two basic problems of elastic stability—(1) the buckling of a rectangular flat plate loaded in compression on two edges and simply restrained along the other two, and (2) the crippling of a hollow cylinder in compression.

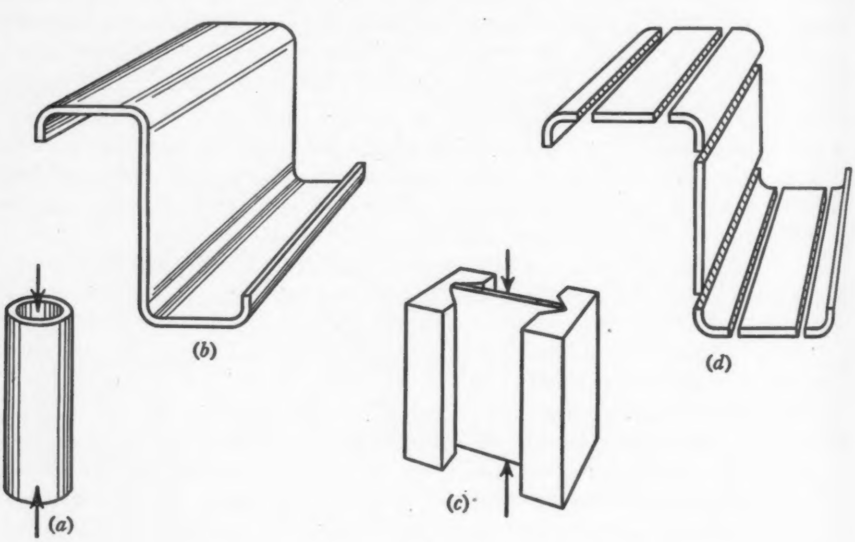

FIG. 2.—STRUCTURAL MEMBERS OF SHEET METAL

Consider a short specimen, consisting of several bends with wide flats between, subjected to an increasing compression load. There is a fundamental difference between the behaviors of the curved elements and the "flats." At low loads, the stress is distributed uniformly. As the load increases, the flats tend to buckle into sinsusoidal waves at some critical stress depending on their width, the widest flat buckling first. These buckles are easily visible to the unaided eye (see Fig. 1). As each flat buckles, it behaves like a miniature Euler column, continuing to carry the load at which it buckled, but going out of action as far as any increase of load is concerned. As the load on the specimen continues to increase, the stress distribution becomes nonuniform, remaining constant in the buckled elements and increasing at a more rapid rate in sections that have not buckled. If the load is removed, the specimen will spring back to its original shape, provided that none of the curved elements have buckled and that the yield stress of the material has not been exceeded. If the load is increased until one of the curved elements buckles, it will crumple sharply. The deformation will be permanent rather than elastic and the

specimen will collapse with any further increase in load. The curved elements do not exhibit any visible distortion until they fail completely.

Summarizing briefly, at low loads the stress distribution is uniform. At higher loads the flat elements buckle, maintaining their critical loads but shedding any increase so that the corners must carry a disproportionate share. Finally, the ultimate strength is determined by the strength of the curved element with the lowest crippling stress. The unevenness of the stress distribution is illustrated graphically by end impressions left on bearing plates used in the tests, where the corners have dug in deeply—whereas the flats have left only a faint impression.

To predict the load at which a short, thin-walled compression member will fail, it is necessary to determine the load that will be carried by each element, flat or curved. The summation of the elemental loads will be the total for the member. The load carried by each element that buckles before final failure will be its area multiplied by its own buckling or critical stress. The load carried by each element that does not buckle before final failure will be its own area multiplied by the lowest crippling stress of any curved element. Since the lowest crippling stress in a curved element is the index of the highest stress that may be carried by the entire specimen, the first problem is to establish the critical stress of that curved element.

S. Timoshenko has established[5] the fact that the crippling stress is a function of the diameter-to-wall-thickness ratio, being higher for low values of this ratio. However, his test data show wide variance from the theoretical curve because, in cylinders of proportions used in practical design, buckling occurs at so high a stress that it is accompanied by yielding of the material. The

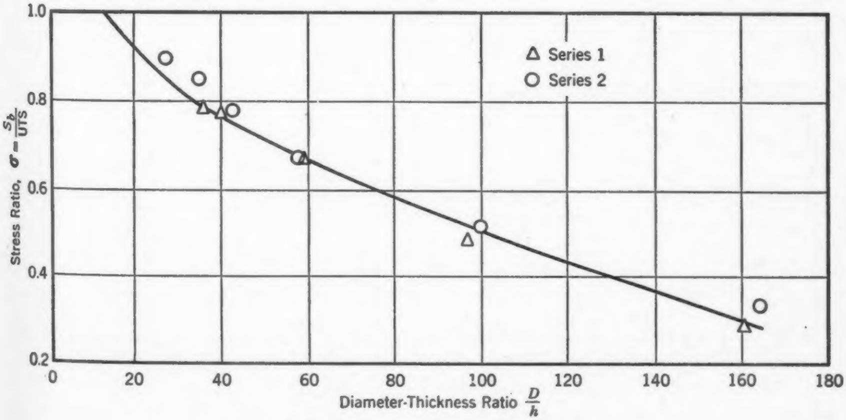

FIG. 3.—CYLINDER CRIPPLING TESTS

problem then becomes one of stability in the plastic range, rather than the elastic range. Instead of attempting to reconcile these complicated phenomena with the theory, it was decided to conduct a test program on hollow, stainless-

[5] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 439.

steel cylinders of various diameters D and thicknesses h, from which the curve of Fig. 3 was determined empirically. The ratios of crippling stress to UTS are taken as ordinates and the ratios of diameter-to-wall thickness are taken as abscissas.

Two series of tests were run. Series 1 consisted of five specimens of constant thickness ($h = 0.017$ in.) but varying from $\frac{5}{8}$ in. to $2\frac{7}{8}$ in. in diameter (D). Series 2 comprised six specimens of 1-in. outside diameter but varying from 0.006 in. to 0.040 in. in thickness. All specimens were $2\frac{1}{4}$ in. long and were made of full hard stainless steel (UTS = 185 kips per sq in.). Welding flanges were trimmed off after welding until they were so narrow that no buckling occurred in them. Each test was conducted in duplicate and the loads were averaged. In no instance did the individual observed values vary from the average by more than 8%. The test results are plotted in Fig. 3.

The purpose of running two series was to detect any influence of the length-diameter ratio. Series 2 (Fig. 3) had a constant ratio of $2\frac{1}{4}$ to 1, whereas in Series 1 there was a range in this ratio from approximately 1 to 1 to 4 to 1. Since all test points fall close to the curve, regardless of the length-diameter ratio, it appears that within the scope of the tests this ratio has small effect.

The critical, or buckling, stress s_b for flat plates, simply supported along two edges, has been derived by Professor Timoshenko,[6] as follows:

$$s_b = \frac{3.62\,E}{(b_f/h)^2} \qquad (1)$$

in which E is the modulus of elasticity; b_f is the width of flat plate; and h is the thickness of plate. Poisson's ratio μ is assumed equal to 0.3.

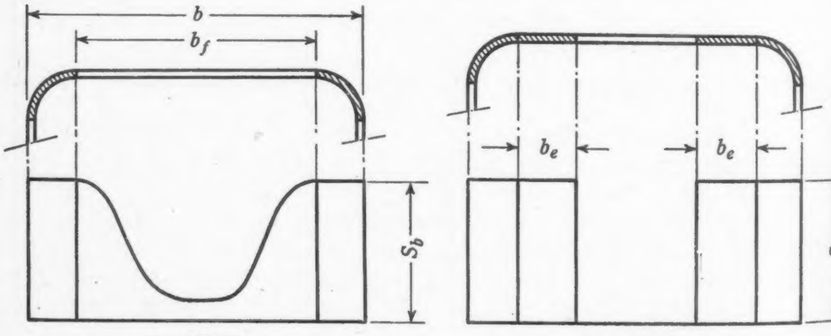

FIG. 4.—COMPARISON OF STRESS DISTRIBUTION

Use of Eq. 1 to find the load carried by a plate after buckling is difficult, since the stress distribution in a buckled plate is not uniform. Rather, it varies from the critical stress at the center of the flat to a stress equal to that of the adjacent material at the edges, as illustrated in Fig. 4. This gives rise

[6] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., p. 329.

to the "effective width" hypothesis,[7] in which a narrow strip along each edge of the plate, working at the same stress as the adjacent round, is assumed to carry all the load in the plate, the stress in the middle segment being zero. The assumption that the central part carries relatively little load is substantiated by many tests,[8,9] which demonstrate that, when the width of a flat plate exceeds certain narrow limits, there is little or no increase in the load it will carry.

Again resorting to empirical means for the development of this method, a series of tests on flat-sided specimens, similar to those shown in Fig. 1, was conducted to determine the effective widths of flat elements. Three series were tested: One group with a constant width of the flat sides equal to $b = \frac{3}{4}$ in. but with thicknesses h varying from 0.006 in. to 0.040 in.; another group of constant wall thickness ($h = 0.017$ in.) but with widths b_f varying from $\frac{7}{16}$ in. to

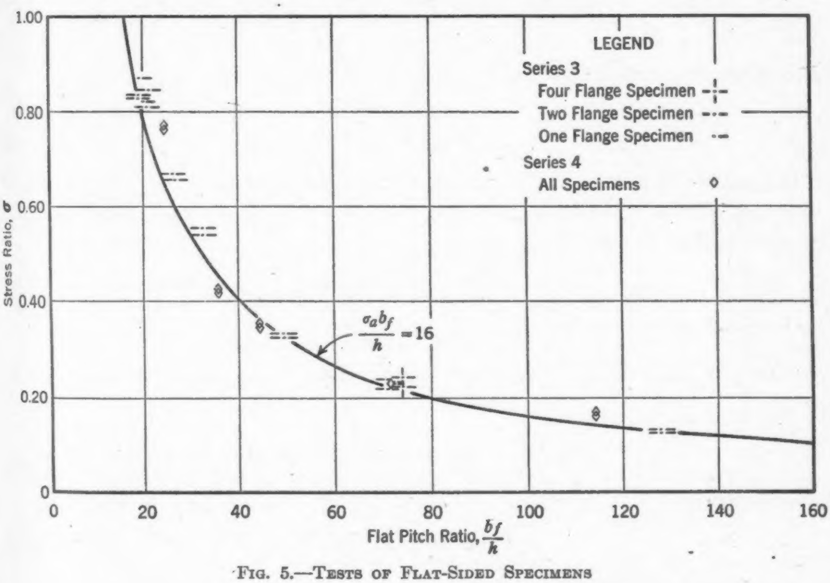

FIG. 5.—TESTS OF FLAT-SIDED SPECIMENS

2 in.; and a third group whose purpose was to evaluate the outstanding welding flanges. These flanges are necessary for the fabrication of the specimens, but their presence affects the validity of the tests. Hence, the part of the load they carry was determined by testing several specimens with different numbers of flanges, but otherwise identical. In the interpretation of test data, only the net areas of the specimens without flanges and the net loads after subtracting the flange loads were considered.

The test results are plotted in Fig. 5. The ratios σ of crippling stress to UTS are taken as ordinates and the flat pitch ratios are taken as abscissas.

[7] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1943, p. 291.

[8] "Strength of Rectangular Flat Sheet In Edge Compression," by L. Schuman and G. Back, *Technical Report No. 356*, National Advisory Committee for Aeronautics, Washington, D. C., 1930.

[9] "The Strength of Thin Plates in Compression," *Transactions*, A.S.M.E., Vol. 54, 1932, p. 53.

It may be seen that the test points are very well expressed by a hyperbola whose equation is:

$$\frac{s_a}{(\text{UTS})} = \frac{16}{b/h} = \sigma_a \text{.......................} (2)$$

in which s_a is the average crippling stress, $\frac{P_t}{A}$; P_t is the crippling load; A is the section area; and b is the gross width of flat, including bend radii.

To be of any use Eq. 2 must be expressed in terms of b_e, the effective width of flat adjacent to each bend, and s_c, the lowest crippling stress of any curved element.

Multiplying Eq. 2 by $b\,h$,

$$\frac{s_a\,b\,h}{(\text{UTS})} = \sigma_a\,b\,h = \frac{16\,b\,h}{b/h} = 16\,h^2 \text{................} (3)$$

Since, with very small error, $s_a\,b\,h = \frac{P_t}{4}$:

$$P_t = 64\,h^2\,(\text{UTS}) \text{.......................} (4)$$

The inside radius of the bends in these specimens was $3\,h$. From Fig. 3 a cylinder with a ratio D/h less than 13 has a crippling stress equal to UTS. The area of four bends is:

$$4\,A_b = 7\,\pi\,h^2 = 22\,h^2 \text{....................} (5a)$$

The load carried by the bends is:

$$4\,P_b = 22\,h^2\,(\text{UTS}) \text{....................} (5b)$$

The net load on four flats is:

$$4\,P_f = (64\,h^2 - 22\,h^2)\,(\text{UTS}) = 42\,h^2\,(\text{UTS}) \text{...........} (5c)$$

Conservatively,

$$P_f = 10\,h^2\,(\text{UTS}) \text{.......................} (6a)$$

but, by definition of effective width,

$$P_f = s_c\,(2\,b_e\,h) \text{.......................} (6b)$$

Therefore,

$$\frac{s_c}{(\text{UTS})} = \sigma_c = \frac{10}{2\,b_e/h} \text{....................} (7)$$

Eq. 7 is plotted in Fig. 6 from which, once the critical curved-element crippling stress is known, the corresponding effective widths of flat elements may be determined.

A flat narrower than $2\,b_e$ will carry a load equal to its own area multiplied by the stress of the bend having the lowest crippling stress:

$$P = b_f\,h\,s_c \text{.......................} (8a)$$

A plate $2\,b_e$ in width, or wider, will carry a load—

$$P = 2\,b_e\,s_c\,h \qquad (8b)$$

—regardless of its width.

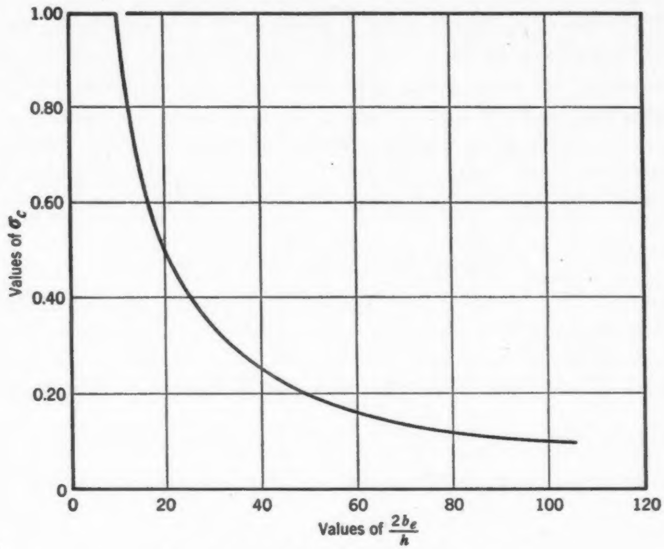

FIG. 6.—FLAT SHEET BUCKLING (EQ. 7)

The general problem of predicting the failing load for the case of local instability of stainless-steel compression members resolves into the following steps:

1. From the curve in Fig. 3 determine the lowest critical stress of any curved element. This will be limited by the curved element with the largest radius.
2. Knowing this stress, find $2\,b_e$ from the curve in Fig. 6. Check all the flats to determine those wider than $2\,b_e$.
3. Add together the areas of all curved elements plus all flats less than $2\,b_e$ in width, plus $2\,b_e$ for each flat $2\,b_e$ or greater in width. The total is the effective area of the cross section.
4. Multiply the total effective area by the stress found in step 1. The product is the crippling load of the member.
5. If desired, the average crippling stress may be found by dividing the crippling load by the gross area:

$$s_a = \frac{P_t}{A} \qquad (9)$$

A combination of three very fortunate circumstances makes possible the further simplification of this method for the great majority of structural members encountered in practice. The first is that, in usual shop practice, a standard bend radius is set up for each metal thickness. The second is that all

these standard radii are small enough so that on the curve in Fig. 3 they fall in the range of D/h in which $\sigma_e = 1.00$. The third is that practically all flats used in practice are wider than $2\ b_e$, which is taken as $10\ h$ for $s_e =$ (UTS).

Then, for one thickness, all 90° bends will have the same radius, crippling stress, and area; hence, they will have the same crippling load. Similarly, all flats will have the same effective area, which is $10\ h^2$ times the same crippling stress, and, therefore, all flats have the same crippling load. It is possible then to tabulate, for each metal thickness used, the crippling load for a flat and the crippling load for a 90° bend.

Using such a table, all a designer needs to do is add up the number of flats, total the number of 90° bends, multiply each sum by the appropriate load from the table, and add the two products. The result is the crippling load for that member.

Example of Use of Method

Consider an 18–8 stainless-steel square tube, similar to that in Fig. 1, with $h = 0.010$ in. and the sides $b = \frac{3}{4}$ in. The UTS is 185 kips per sq in. Assume the length L to be small, so that column failure is precluded. It is desired to predict the load at which crippling will occur. The inside radius of all bends is 3/64 in.; and $D/h = \frac{2}{0.010}\left(\frac{3}{64} + 0.010\right) = 11.4$. From Fig. 3, when $D/h = 11.4$, $\sigma_e = 1.00$ so that, in this example, $s_e = 185$ kips per sq in. From Fig. 6, when $\sigma_e = 1.00$, $\frac{2\ b_e}{h} = 10$. Then $2\ b_e = 10\ h = 0.10$ in. All the sides are considerably wider than this.

Outstanding flanges are arbitrarily evaluated at half the value they would have if supported on two edges. This is known to be incorrect scientifically, but their proportionate area is small and very little error is introduced.

The total effective area for this specimen comprises two 90° bends, four 45° bends, and four flats and four flanges. By tabulation:

Description	Computation	A
Say, four 90° bends	$0.104 \times \pi \times 0.010$	0.00327
Four flat sides at $2\ b_e\ h$ each	4×0.001	0.00400
Four flanges at $b_e\ h$ each	4×0.0005	0.00200
Total area, in square inches		0.00927

and $P_t = 0.00927 \times 185{,}000$ lb per sq in. $= 1{,}715$ lb.

The average crippling load of several test specimens answering this description is 1,850 lb. The test specimens were actually 0.0104 in. thick and had an ultimate tensile stress of 195 kips per sq in. Roughly correcting: $P_{test} = 1{,}850 \times \frac{185{,}000}{195{,}000} \times \frac{0.010}{0.0104} = 1{,}690$ lb, against 1,715 lb computed.

Care must be taken in applying this method to members subject to other forms of instability. For example, long slender members may fail as columns. Open sections—that is, members whose sides do not form a closed tube—are subject to torsional instability unless well restrained to prevent twisting.

Another manifestation of instability to be guarded against is that illustrated by a square or round tube whose walls are formed of corrugated sheet, the corrugations running longitudinally. In such a member there would be no flat surfaces, and at first glance the over-all strength might appear to be the summation of strength of all the curved sections. The corrugated sheet, however, is subject to a general instability of its own, which may be thought of as a column failure of the corrugations.

Extension of the Crippling Method to Compression of Members That May Fail as Columns

The use of the foregoing simplified method of computing the failing load under conditions of local instability is limited to very short compression members. It is obvious that, in extremely long compression struts, failure resulting from primary column instability will occur at a stress so low that there will be no local buckling. In quite a wide range between these two extremes of length, failure may occur as a combination of local and primary instability, each contributing to the other. Local buckling means reduced effective area, reduced moment of inertia, and as a result—reduced radius of gyration. On the other hand, column curvature results in unequal stress distribution, enhancing the buckling on the side where the compression is increased.

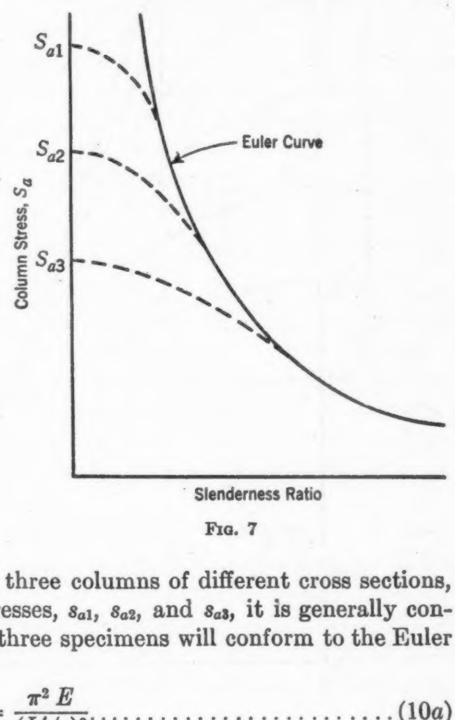

Fig. 7

Because a rigorous treatment of these complex interactions is tedious and complicated, semi-empirical methods will again be adopted. Referring to Fig. 7 in which critical column stress is plotted against slenderness ratio for three columns of different cross sections, having different average crippling stresses, s_{a1}, s_{a2}, and s_{a3}, it is generally conceded that for very long columns all three specimens will conform to the Euler equation,

$$\frac{P}{A} = \frac{\pi^2 E}{(L'/r)^2} \qquad (10a)$$

It is also evident that the limiting stress for each at very small values of (L'/r) will be its own crippling stress, s_a.

Arbitrarily treating the average crippling stress as the column yield stress and plotting for each case the Johnson parabola—

$$\frac{P}{A} = s_y - \frac{s_y (L'/r)^2}{4 \pi^2 E} \qquad (10b)$$

in which s_y is the column yield stress, assumed equal to s_a—the dotted curves in Fig. 7 are obtained. The Johnson formula is no more justified scientifically than are other short column formulas; that is, it agrees with tests made over a period of years.

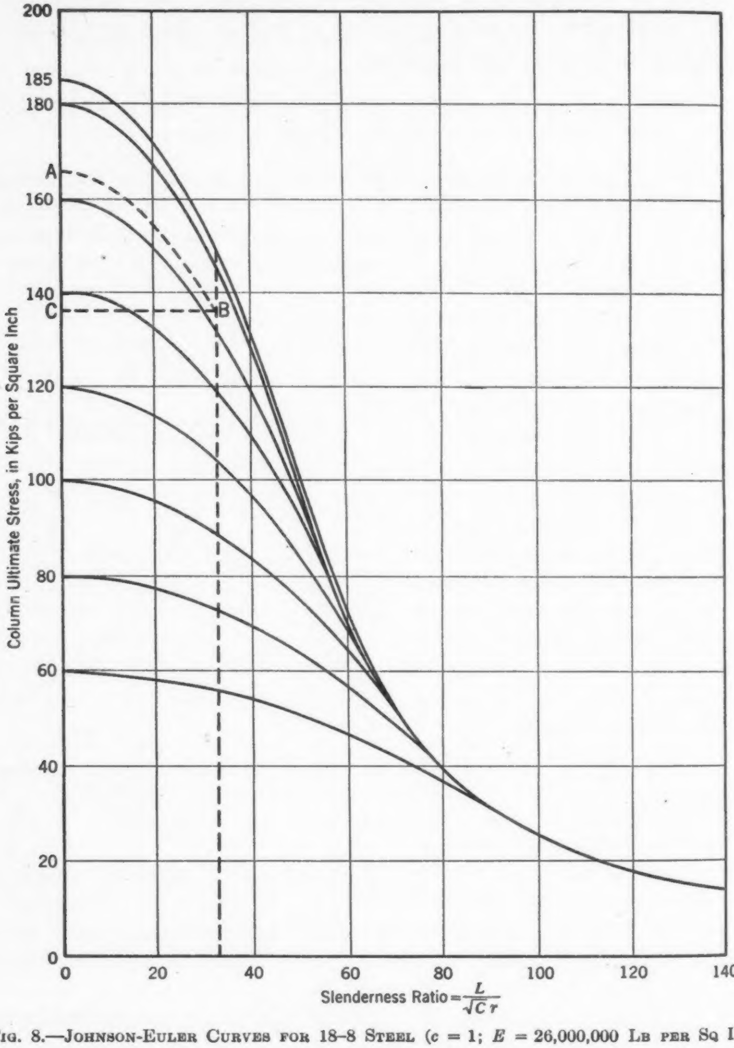

Fig. 8.—Johnson-Euler Curves for 18-8 Steel ($c = 1$; $E = 26{,}000{,}000$ Lb per Sq In.)

Finally, the chart in Fig. 8 was made by plotting a family of Johnson-Euler curves starting at regular intervals of column yield stress.

The procedure in using the chart is as follows:

(*a*) Determine crippling load by the method previously described.

(*b*) Divide crippling load by gross cross-section area, to obtain average crippling stress (Eq. 9).

(*c*) Beginning with this crippling stress on vertical scale (point A, Fig. 8), follow the curve down to intersection with appropriate slenderness ratio (point B, Fig. 8).

(*d*) Read horizontally across to find ultimate column stress (point C, Fig. 8).

In a large number of tests, in addition, and subsequent to the tests by which the method was derived, predictions have been confirmed within the limits of the variations in thickness, hardness, workmanship, and all other factors involved.

This method has been a tremendous labor saver in the design and stress analysis departments and has been substantiated by experience with a number of structures which have stood up satisfactorily in static tests and in service.

Conclusions

The stress at which thin plates in a structure buckle is not necessarily the limit of useful strength. Compression members having wide, thin plates—particularly those built up from sheet metal—may have an ultimate strength considerably greater than the load at which first buckles appear.

The concept of an "effective area" is used to compensate for the reduced efficiency of the elements that have buckled. Factual data, derived from a test program, are presented for 18–8 stainless steel in the form of curves, from which the effective area and crippling stress may be obtained. The curves are strictly applicable only to stainless steel, but the method of their derivation may be applied to other metals.

Experience in the use of this method leads to the realization that there is much more work yet to be done. To predict the behavior of plates restrained on only one edge, "pile-ups" of several metal thicknesses spot-welded together, and numerous other complex arrangements encountered in practical design, still requires considerable judgment. Care must be taken not to overlook other forms of instability such as general column stability of the entire member, or the stability of loosely connected parts of a member, and torsional instability of open sections.

Within its limitations, the method described has proved to be a rapid and reliable method of predicting the crippling strength of stainless-steel compression members.

Acknowledgment

The writer is indebted to many among the technical personnel of the railway and aircraft divisions of the Budd Company, Philadelphia, Pa., who contributed to the development of this material.

APPENDIX. NOTATION

The following letter symbols, defined where they first appear in the text, or by illustration, conform essentially with the proposed American Standard

Letter Symbols for Structural Analysis prepared by a Committee of the American Standards Association, with Society participation:

A = cross-section area; A_b = cross-section area of a bend;
b = breadth or width of a plate; gross width of flat, including the bend radii:
  b_e = effective width of the flat part adjacent to each bend;
  b_f = the flat part of the breadth;
c = coefficient of fixity;
D = outside diameter of a hollow cylinder;
E = modulus of elasticity;
h = wall thickness of stainless-steel sheet metal;
L = length of specimen; L' = reduced column length, $\frac{L}{\sqrt{c}}$;
P = concentrated load; total load:
  P_b = part of the load P carried by a bend;
  P_f = part of the load P carried by a flat;
  P_t = crippling load; entire member;
r = least radius of gyration;
s = unit stress (ultimate tensile unit stress, UTS):
  s_a = average crippling stress;
  s_b = buckling stress, flat element or plate;
  s_c = critical stress, curved element or cylinder;
  s_y = yield stress;
μ = Poisson's ratio; and
σ = stress ratio:
  $\sigma_a = s_a$/UTS;
  $\sigma_c = s_c$/UTS.

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DISCUSSIONS

CONTINUOUS FRAME ANALYSIS BY ELASTIC SUPPORT ACTION

Discussion

BY J. CHARLES RATHBUN AND C. W. CUNNINGHAM

J. CHARLES RATHBUN,[64] M. ASCE, and C. W. CUNNINGHAM,[65] Assoc. M. ASCE.—The large number of discussions of the paper is gratifying to the writers, as is also the fact that many discussers have made distinct contributions to the problem of frame analysis. In some of the discussions other methods of solving the same problem are presented by those who prefer them to the one presented in the paper. Although such citations are not pertinent to the subject, they do, in most cases, contribute to the problem of the solution of the rigid frame by calling attention to these methods and are, therefore, of value.

The first paragraph of the "Synopsis" states clearly that the moment-distribution method is entirely distinct from the one presented. The words "classical" and "exact" have been used to show that the method is not one of successive approximations. The writers' use of the word classical is probably a survival of Newton's day, when the method of successive approximations was distinct from the classical methods (the Greek and Roman traditional methods).

An exact method, as used in the paper, is one which derives an exact solution from the data given. A method of successive approximations cannot be exact, in general, although it may be correct or close enough to the exact solution to satisfy the requirements. As an example, π has been computed to more than seven hundred decimal places, but this is not an exact value. Neither classical nor exact is used here in a sense complementary, or otherwise, to any method.

Of the seventeen discussions nine compare this method with the well-known moment-distribution method. These discussions are of little value because

NOTE.—This paper by J. Charles Rathbun and C. W. Cunningham was published in April, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by L. J. Mensch, Frederick S. Merritt, I. Oesterblom, Thomas C. Kavanagh, A. Floris, and Tao King; November, 1947, by Leroy A. Beaufoy, A. A. Eremin, Robert B. B. Moorman, Eduardo Agramonte, Stephen J. Fraenkel and Robert L. Janes, and Phil M. Ferguson; January, 1948, by Ralph W. Stewart, and Maurice Barron; and March, 1948, by Edwin H. Gaylord, William A. Conwell, and Harry Garfinkel.

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[65] Associate Prof., Civ. Eng., College of the City of New York, New York, N. Y.

they do not apply to the method of the paper or contribute to the subject. Moment distribution is a most valuable method when the answer to a specific problem is desired, but many engineers are willing to consider that it furnishes all that is required for a solution, and they refuse to consider attempts to improve conditions.

Mr. Beaufoy furnishes an example of the trial-and-error solution in Fig. 15, wherein he compares this method with that of the paper. In Fig. 15 there is a self-consistent group of values, but moment EG is in error by 3.67%, or an amount equal to 0.425, and the error in moment GH is equal to 0.698. In his discussion, Mr. Beaufoy does not indicate a way to estimate the amount of these errors. In many problems one may obtain a solution by an approximate method with fewer computations and in less time than by an exact method, but does that constitute a solution to the general problem? The writers think not, and, although they admire this method for practical solutions, they consider it to have no place in a discussion of the paper. This same principle could have been applied in solving for the moments at the fixed joints, since the convergence of the series of trials will usually be very rapid. As this is quite obvious, the writers purposely omitted it from the paper.

The method of Otto Gottschalk is based on unloaded models. It is, therefore, useful only as a check on any analytical solution.

Messrs. Barron and Garfinkel advocate the use of transmission coefficients; this method requires a preliminary assumption for the value of a transmission coefficient. Although the assumed value produces final results that are within the limits of practical accuracy, the method is not exact, and so is not applicable to the writers' method.

Contrary to the statements of Messrs. Barron and Garfinkel, vertical and horizontal sway were treated simultaneously. If there are unbalanced shears in two mutually perpendicular directions, and if these shears are balanced in one direction, those in the other direction are automatically balanced—provided that all joints are in equilibrium. The following is a proof of this statement as applied to Example 5:

$$M_{EA} + M_{AE} + M_{FB} + M_{BF} = 24 \qquad (117a)$$

and

$$M_{AC} + M_{CA} + M_{BD} + M_{DB} = -24 \qquad (117b)$$

Adding Eqs. 117*a* and 117*b*,

$$+ M_{EA} + M_{AE} + M_{FB} + M_{BF} + M_{AC} + M_{CA} + M_{BD} + M_{DB} = 0 \qquad (117c)$$

Substituting from the equations for joint equilibrium,

$$- M_{EF} - M_{AB} - M_{AC} - M_{FE} - M_{BA} - M_{BD} + M_{AC} - M_{CD} + M_{BD} - M_{DC} = 0 \qquad (117d)$$

—which reduces to

$$M_{EF} + M_{FE} + M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0 \qquad (117e)$$

—showing that the sum of the shears in the vertical members is zero, as it should be.

Mr. Barron questions the extension of the method as stated. If the principles outlined are followed, the method can be applied to any type of continuous frame having members of constant or variable moments of inertia. The question of secondary moments in Vierendeel trusses due to change of length of the members caused by the axial stresses, as well as the effect of the width of columns and depths of beams on the analysis, is definitely outside the scope of this paper.

Mr. Oesterblom doubts the adaptability of the method to Example 7—a complicated frame with various types of loading. The authors have solved this problem, obtaining the numerically correct answer, consistent with the assumed data. This work can be duplicated by following the paper and the given examples. The validity of the method and its proof have been brought out fully in the paper. Mr. Oesterblom does not make it clear why he cannot accept it.

Professor King claims an advantage for moment distribution in computing joint and bar rotation. If there is no joint translation, Eq. 9*b* may be used to obtain joint rotation. Eq. 18 may be used to find the joint rotation if there is joint translation, noting that the shear correction factor is equal to $\frac{E\,\Delta}{L}$. Calculations of these quantities by the elastic support method are much simpler and more direct than by the method advocated by Professor King.

Messrs. Fraenkel and Janes (and others) seem to have missed the reason for presenting simple illustrative problems such as the first three. There is no doubt that the first example can be solved more readily by the three-moment theorem; however, the purpose of the examples was to familiarize the reader with the method. There has been criticism of the use of the "conjugate beam" method in deriving some of the formulas. Other accepted methods of deriving these formulas have been used, but it was thought that the conjugate beam method was the best known to the readers.[66]

Mr. Merritt has used the method of moment ratios to solve and check several of the examples of the paper. His expression of "one-cycle distribution" is very apt, and brings out one difference between the moment-distribution method and some newer methods. The preliminary computations should be reduced to a minimum for practical considerations, but, when a new method is being presented, clarity should not be sacrificed for brevity. In the solution of Example 4 Mr. Merritt assumes two moment ratios, which remove his method from the "theoretically exact" category. Both the methods of Mr. Merritt and R. C. Brumfield, M. ASCE, have features that contribute to the ultimate solution of the problem.

Mr. Kavanagh has made a valuable contribution by transforming Eqs. 8 and 11 into one that is well known in the literature of elastic stability of bars, thus indicating the possibility of applying the method to the calculation of buckling restraints in axially loaded members of rigid frames.

Mr. Eremin presents a graphical construction that should be very useful in checking the analytical work for both prismatic and nonprismatic members, and so is an addition to the subject.

[66] "Elastic Properties of Riveted Connections," by J. Charles Rathbun, *Transactions*, ASCE, Vol. 101, 1936, pp. 550–552.

Professor Moorman's equations for the statical moment of the moment diagram for prismatic members can be used, not only in this method, but also in any method involving these quantities. The evaluation of $\bar{x}$ usually presents no difficulty if one sketches the moment curve of the simple beam and then by moments (as in the conjugate beam method) finds its center of gravity.

Mr. Agramonte has developed an interesting but formidable set of equations for nonprismatic members. However, it should be noted that the θ used by Mr. Agramonte is measured from the rotated position of the gravity axis of the member (Fig. 22), whereas the writers measure θ from the position of the unstressed member (Fig. 5 (*b*)).

Professor Ferguson calls attention to the method presented by T. Y. Lin, Assoc. M. ASCE; he then points out the difference between the two papers, noting that the formulas differ, but that one set can be reduced to the other by algebraic transformations. This should be true if both sets of formulas are correct and if they deal with the same ideas. If, as Professor Ferguson thinks, a comparison between the two methods is valuable, it would have been a contribution if he had worked it out in his discussion. He mentions several differences, enough to make it evident that the Lin method is not the same as that of the writers, but that it is valuable.

Professor Gaylord has simplified the writers' equations in a very ingenious manner which ought to appeal to the practicing engineer. Eq. 93*h* is particularly significant since it shows how fixed-end moments are distributed about a joint that is temporarily fixed, and then allowed to assume its natural rotated position.

Mr. Conwell's remark about the analytic beauty in the concept of the method presented is appreciated. Inclusion of the properties of the connections in the analysis is, the writers feel, a must. For work to be of value in practical stress analysis there are usually other considerations besides elasticity of connections that affect the answer even more. For example, a building is stiffened 350%, or thereabouts, by the walls, partitions, and floors. From a strictly practical standpoint there is some question as to the value of any analytical method. However, this does not mean that progress should stop in the development of these methods.

The writers do not agree that the final test is in the designing office, where the designer may not have the time, energy, or desire to learn a new method. It is their experience that those who have taken time to develop skill in its use find that the analysis by elastic support action is as rapid, or more rapid, than the other methods. Its chief value is, however, the presentation of a method which will further the subject of analysis. If advancement were to stop with any of the current methods it would indeed be unfortunate.

Mr. Stewart shows considerable similarity between some of the writers' ideas and the method of traversing the elastic curve; this method is probably more popular than is generally supposed. Combining as it does the advantages of geometry and pictorial representation with the moment-area method, a very efficient system has been devised. Not only is it simple and readily understood, but problems can be solved quickly, and, because of the picture, the action is understood. As previously stated, the writers chose the conjugate beam

method of deriving their equations, rather than the one suggested by Mr. Stewart, because they are of the opinion that the older method is more widely known.

Mr. Floris' statement that the writers' method is indirect, that it is primarily a relaxation method, and that conjugate beam is an incorrect name for the Mohr theorem (moment-area method or Greene's theorem) leads the writers to feel that he has deviated widely from the proper scope of discussion. That proof of the formulas and method presented has not been furnished is an unwarranted statement; Eq. 47 is the same as Eq. 10*b* with a slight, unnecessary, algebraic change, which makes it a little difficult to use. Mr. Floris' concluding paragraph is his opinion against that of the writers, whose experience has been to the contrary. Since his discussion shows some misunderstanding of the subject, he should have supported his conclusions so that they could have been of value.

In closing, the authors wish to thank those discussers who have taken the time and trouble to help develop the ideas presented, and to call attention to the writings of others along these lines.

Corrections for *Transactions*.—In April, 1947, *Proceedings*, page 419, Fig. 5(*b*), the lower dimension arrow for θ_A should point to the tangent line. In January, 1948, *Proceedings*, page 106, Table 20, in the "Member" column, reverse the order, thus: BA should be AB, etc.; on page 109, Table 21, again in the "Member" column and in the "Moment split" column reverse the order of each designation; and, on page 114, add the following insert before the last paragraph:

> "Figs. 30(*a*) to 30(*f*) show various types of rectangular and nonrectangular frames. Fig. 30(*g*) shows three curved members with a common point A, and Fig. 30(*h*) shows a frame with both curved and straight members. In Fig. 30(*j*), the intrusion of the column into the ordinary building beam is unimportant, but, in Fig. 30(*k*), the column intrusion is a large part of the beam span and must be considered in the analysis."

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DISCUSSIONS

THE GENERAL PRINCIPLES OF HIGHWAY TRANSITION CURVE DESIGN

Discussion

BY RALPH L. FISHER, WILLIAM R. WELTY, ELMER R. HAILE, JR., AND CHARLES M. NOBLE

RALPH L. FISHER,[16] ASSOC. M. ASCE.—A realistic approach to the problem of the transition curve has been presented by the author, who states that "vagueness is inherent to the subject" and cumbersome "mathematical treatment is not justified by the facts."

The basic function of a spiral curve is to enable a driver to develop the necessary steering angles (and friction values) with "safety and comfort" within the limits of his lane. On a tangent or on a circular curve a driver is subject to constant, known conditions and it is only on the spiral that he must make adjustments. It is awkward to drive on curves that are transitional throughout. Even when traveling on a tangent, a driver is constantly moving the steering wheel in order to keep on his path. On a curve of constant radius these adjustments in steering are also necessary. If the radius of the curve is constantly changing, the driver's task is made more difficult.

Mr. Leeming suggests that curves have from one half to two thirds of their length transitional. Except for the rare case of equal length curves, this would make every spiral different, even those having the same radius of circular curve. If spirals replace a large part of the circular curve, they must perform a function for which they are not suited, that is, change in direction. For this latter function, the circular curve, with its constant conditions, is best fitted. It would be better design practice to confine the spirals to each end and thereby be able to use a larger radius for the circular curve, thus creating more uniformity of operating conditions.

The alinement, grades, superelevation, and spirals should be designed as a unit. Too often the spirals are added more or less as an afterthought, with the result that the spirals or the superelevation runouts are at bridges, vertical curves, minimum grades, or other unfortunate locations—thus creating prob-

NOTE.—This paper by John Joseph Leeming was published in October, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1948, by Carl F. Meyer.

[16] Engr. of Design, State Highway Dept., Trenton, N. J.

lems that need never have occurred. Any transition curve should provide a path conforming to driver behavior. Observation indicates that most drivers tend to "cut corners" and use more throw than is provided by the cubic spiral. The designers of the Pennsylvania Turnpike developed a 2.5 power spiral, which gives more throw than the cubic spiral. It was not used because of lack of time to make the necessary field tests and because of lack of tables for the field forces.

The writer agrees with Mr. Leeming that (within limits) the length of the transition is not a matter of great importance. As he states, there is considerable variation in a driver's practice, even on the same curve and at the same speed. For the same conditions, a driver does not develop the steering angles at the same rate. It is believed that the amount of throw is of more importance than the rate of curvature of the transition spiral.

Before decision as to the lengths of the spirals can be made, the method of obtaining the superelevation must be determined. This requires careful study for multilane roads or divided roadways with narrow islands. In the colder regions consideration must be given as to where snow can be ploughed on the superelevated sections. The cross section should be designed so that during periods of thawing and freezing, water from the melting snow will not run on the pavement and freeze, thus becoming a hazard. At sags, care should be taken that the low side does not go below certain minimum elevations for surface drainage or water table conditions. Detailed studies may have to be made at bridges where vertical clearance is limited.

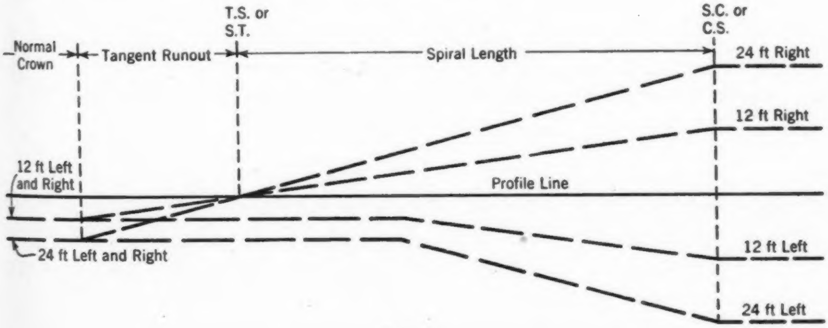

Fig. 4

For the foregoing reasons, the length of spiral is rarely governed by the distance necessary for change in centrifugal acceleration, as distances given by this method are generally too short. With the flat grades used on most highways today, the length of spiral is generally governed by the minimum gutter grades. If the lengths of the spirals and tangent runouts are selected to fit these flat grades, they can be used throughout, and there will not be the necessity of using one length for the steeper grades and having "special cases" for the flatter grades.

Fig. 4 shows the usual method of obtaining superelevation with spirals. In cases of 5% superelevation, a spiral 400 ft long is required if the relative

slope of the outer edges of the pavement with respect to the center line is held to a change of 0.3%. With a center line grade of 0.5% this means that one edge of the pavement would have a grade of 0.2%. However, the shoulder can be warped to give minimum gutter grades of 0.3%.

Uniformity of operating conditions is the keynote of modern highway design. In practice, the design speed is usually selected first and then the other elements, including transition curves, are designed to be consistent with this design speed. As an example, with an assumed design speed of 60 miles

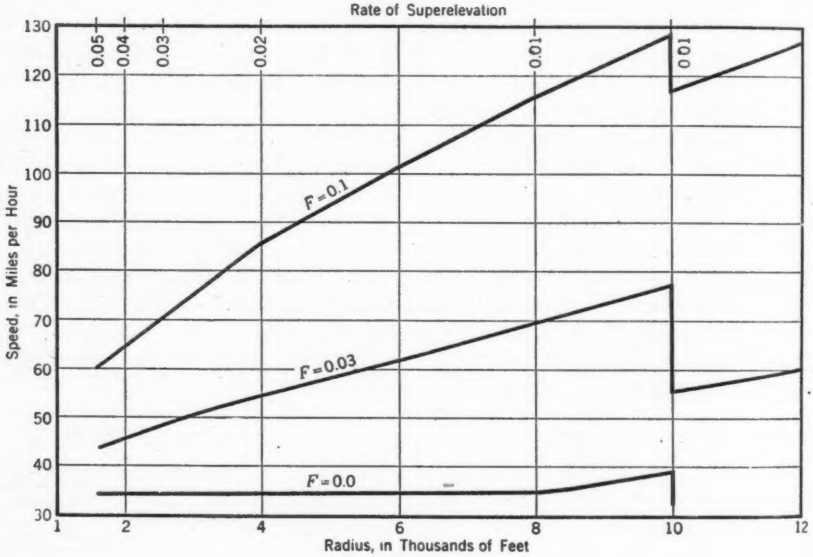

Fig. 5

per hr and a minimum radius of 1,600 ft, for a friction factor of 0.1, E would equal 0.05 from

$$V^2 = \frac{R\,E'}{0.067} \qquad (14)$$

For other radii, the superelevation varies inversely as the radius.

Fig. 5 shows the loci for $F = 0.0$ (balanced forces), $F = 0.03$ (assumed icy surfaces), and $F = 0.1$. The locus for balanced forces indicates uniform and consistent operating conditions and the speed is low enough for most trucking units. The break in the curves at the 10,000-ft radius point is due to the change from a superelevated section to a normal crowned section.

For the cubic spiral, the degree of curve and the superelevation varies directly with the length from the point of tangency (T.S.). Table 4 shows this relationship for a 400-ft spiral to a curve of 1,600-ft radius with 5% superelevation. The curvature of this spiral varies from infinity to 1,600 ft, and this curvature is fixed. It seems apparent that, if this rate of curvature is suitable for a transition to the minimum radius, it would also be suitable for curves of

larger radius if the proper length of the sharper end were deleted. In other words, by selecting a proper segment of this spiral to meet a given radius, it would be suitable for all curves of 1,600-ft, or greater, radius. This seems to be more logical than having a spiral with a different rate of curvature for each radius.

TABLE 4.—RATE OF SUPERELEVATION FOR 400-FT CUBIC SPIRAL TO CURVE OF 1,600-FT RADIUS HAVING 5% SUPERELEVATION

Distance from T.S. (ft)	Rate of superelevation (%)	Radius (ft)	Distance from T.S. (ft)	Rate of superelevation (%)	Radius (ft)
(1)	(2)	(3)	(1)	(2)	(3)
0	0	∞	250	3.12	2,560
50	0.62	12,900	300	3.75	2,130
100	1.25	6,400	350	4.37	1,830
150	1.87	4,280	400	5.0	1,600
200	2.50	3,200			

Fig. 6 was prepared from Table 4. This series of curves gives a throw of 6.40 ft for the 1,600-ft radius, compared with a throw of 4.17 ft for the cubic spiral. This additional throw is believed to conform more nearly to driver behavior. However, if desired, by changing the radii slightly, these curves

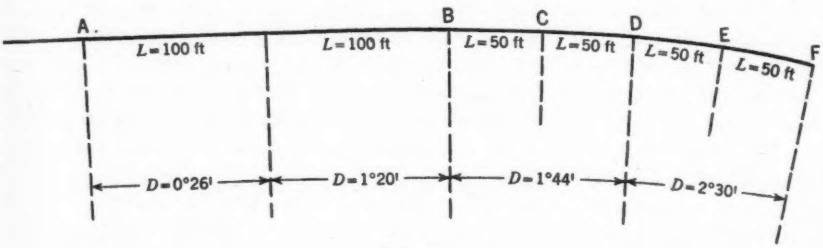

FIG. 6

could approximate the cubic spiral very closely. The transitions from point A to point B would be used for curves of 3,500-ft radius to 3,200-ft radius; from A to C, for 3,200 ft to 2,500 ft; from A to D, for 2,500 ft to 2,100 ft; from A to E, for 2,100 ft to 1,800 ft; and from A to F, for 1,800 ft to 1,600 ft. In actual practice, it would probably be necessary to develop two series of transition curves, one for general conditions and another of minimum length for cases where the middle ordinate or tangent lengths are restricted. All elements of these curves as required by local surveying practice could be easily computed, thus doing away with the necessity for any tables or computations by the field parties. This is a factor to be considered, particularly during periods when most highway departments are experiencing a shortage of trained personnel.

WILLIAM R. WELTY,[17] Assoc. M. ASCE.—It is gratifying that the research of the English engineering profession in this hereto virtually unexplored subject is made available to the American engineer in this fine paper. The fact that

[17] Traffic Designing Engr., State Highway Dept., Austin, Tex.

transition curves in the past have been designed on fallacious assumptions does not require that the same procedure continue in the future.

The writer had the opportunity to examine the spiral experimentally, using the photographic method developed by Bruce D. Greenshields,[18,19] Assoc. M. ASCE. This study [20] of the transition curve was instituted by the writer due to recurring doubts as to the validity of the conventional spiral transition theory. The work of Kenneth A. Stonex and Charles M. Noble,[21] M. ASCE, H. A. Warren,[8] and Mr. Leeming and A. N. Black,[9] certainly casts doubt on the preconceived theories, if it does not entirely shatter them. In the study using the photographic method, a specially arranged motion picture camera and timer were set up at a high vantage point (for example, on a skycraper or in a blimp) above and close to the curve to be studied. The camera was arranged so as to take one picture every eighty eighth of a minute. Thus, the motion of the vehicle in feet between successive frames gives the vehicle's velocity in miles per hour. For examination of the vehicle's path, its position was plotted to scale for each 1/88-min interval. The points at the approximate center of the curved path were fitted graphically with a circular arc of the smallest radius which would pass through the greatest number of points. The length of transition used by the driver was then the distance between his former straight path and the circular one. Since each plotted point represented a 1/88-min interval, the time spent in the transition was readily computed. To determine W. H. Shortt's symbol *"C"* or Mr. Leeming's symbol *"Q"*, we may, by making an assumption that the rate of change of lateral acceleration *"Q"* is a constant for any path of a vehicle over the transitional length of the curve (verified by Messrs. Leeming and Noble), easily compute *"Q"* by dividing the centrifugal force minus the superelevation by the time spent in the transition.

The study, although of very limited scope and by no means comprehensive, did seem to bear out Mr. Leeming's conclusion that the value *"Q"* is by no means a constant. A direct relationship seems to exist between *"j"* and Q; Q seemed to have a value between $31.4\,j$ and $47.2\,j$. Measurements were not made of high speed operation, but some values measured with an accelerometer on the Pennsylvania Turnpike by Messrs. Stonex and Noble approximately fitted the graphical relationship.

According to the Shortt theory of a constant C or Q, the length of transition will vary with the cube of the velocity. It was found in the photographic study that for the same curve the faster drivers used the shortest transitions, which is in opposition to the former theory. The writer believes that the reason the faster drivers use a shorter transition than do the slower drivers may be explained in the following manner:

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[18] "Photographic Method of Studying Traffic Behavior," by Bruce D. Greenshields, thesis presented to the University of Michigan at Ann Arbor, Mich., in 1934, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[19] "Traffic Performance at Urban Street Intersections," by Bruce D. Greenshields, Donald Schapiro, and E. L. Erickson, *Technical Report No. 1*, Yale Bureau of Highway Traffic, New Haven, Conn., 1947.

[20] "A Method for Studying the Paths of Motor Vehicles on Curves," by William R. Welty, Bureau of Highway Traffic, Yale University, New Haven, Conn., May, 1946.

[21] "Curve Design and Tests on the Pennsylvania Turnpike," by Kenneth A. Stonex and Charles M. Noble, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 20, 1940, p. 429.

[8] "Experimental Transition Curves" by H. A. Warren and E. R. Hazeldine, *Journal*, Inst. Municipal & County Engrs., London, March, 1939, p. 1021.

[9] "Road Curvature and Superelevation: Experiments on Comfort and Driving Practice," by J. J. Leeming and A. N. Black, *ibid.*, December, 1944, p. 137.

1. Fast drivers expect on entering a curve to develop considerable side thrust.

2. Since the side thrust is uncomfortable to the driver (and he expects that it will be), he develops the maximum side thrust quickly, continues it only as long as is necessary, and gets out of the curve as soon as possible.

3. On the other hand, the slow driver, since motorists are humanly lazy, turns the steering wheel at a minimum rate to save work, thus using a long transition. A long transition does not, in this case, prolong the discomfort of a high value of side thrust, since at slow speeds a high value of side thrust is not reached.

The accelerometer method of measurement possesses certain advantages: First, the rate of change of lateral acceleration may readily be determined; and, second, the transitional and circular lengths of the path are easily secured. On the other hand, it is believed, as recognized by Mr. Leeming, to be quite possible that the fact the driver knows his actions are being recorded, in the case of the accelerometer method, may cause significant changes from his normal behavior. By use of the photographic method, measurements may be made without the driver's knowledge.

Elmer R. Haile, Jr.,[22] Assoc. M. ASCE.—Although his argument may, as the author states, run counter to established practice, it is in general agreement with observations made by the writer several years ago. At that time the writer measured the rate of turning of the steering wheel on entrance to a curve, and concluded that no definite trend could be discerned because of wide variations in the rate observed during several test runs. Mr. Leeming is to be commended for carrying his experimentation and analysis through to logical conclusions.

The writer agrees with the author's four conclusions (under the heading, "6. Conclusions from the Experiments"). He does not agree, however, that the proportion rule is applicable in all cases, and this discussion is intended to point out certain limitations in its use. With reference to the third conclusion, the writer believes that the tendency for drivers to use from two thirds to one half of the curve as the transition is due to an inherent characteristic of motoring that is not generally recognized. This might be called the characteristic of "smoothing" the path of the vehicle.

Accepted design theory is based on the assumption that the vehicle behaves like a locomotive fixed on rails, whereas, in reality, each driver selects the path for his vehicle, and constantly tends to smooth out any "kinks" in the road. This action is usually subconscious and, in fact, unavoidable, stemming from the fact that the driver guides his vehicle by the appearance of the road many car lengths ahead. In contrast, the railroad locomotive is guided solely by the rails through the pony wheels. The inability of the driver to see the road at the front wheels is occasionally embarrassing—for example, when attempting to drive up a ramp or onto a hydraulic lift at a service station. However, in high speed driving on the highway, this inability to see the front wheels is comfortable rather than uncomfortable.

[22] Highway Engr., U. S. Public Roads Administration, Arlington, Va.

Some examples of this tendency of drivers to smooth the courses of their vehicles are as follows:

(1) In making a right turn at a street intersection, drivers tend to swerve left before turning right, and also to swing wide into the cross street in order to obtain a longer radius of curvature. This behavior is often observed, and persists in spite of many collisions and near collisions that result from it.

(2) A circular curve, when viewed from a distance, appears to break sharply from the tangent. There is a tendency to smooth out this break in the same manner as described in item (1). This smoothing is better known as the transition.

(3) An all-transitional curve, say, with a length of 300 ft and a central radius of 250 ft, appears to bend sharply in the middle. Drivers invariably traverse the middle of such a curve with a radius longer than the design radius, and, if traffic permits, they fail to confine their vehicles to the proper traffic lane.

(4) A short curve with a small deviation angle appears to be a bad break when it is located between two long tangents. Drivers invariably "cut the corner," particularly at high speeds.

In the foregoing examples, the underlying motivating force is the aim of the driver to select a smooth path between the location of the vehicle at any instant and some point ahead that appears in the field of vision. In slow city driving, the driver may gage his path to a point less than 100 ft ahead, but in fast rural driving he may keep his eyes on the road several hundred feet ahead.

The properties of two curves computed by conventional methods are compared with curves computed by the proportion rule, as follows:

Conventional Method.—

Curve 1.—Assume that $v = 40$ miles per hr; $\Delta = 35°$; and that the tangents are limited. Then the minimum allowable $R = 409$ ft and $L = 250$ ft, using a maximum superelevation of 0.10 ft per ft.[23] Then $\phi = 17° 30'$; $T = 256$ ft; and $E = 27$ ft.

Curve 2.—Assume that $v = 60$ miles per hr; $\Delta = 21°$; and the tangents are limited. Then the minimum allowable R = 955 ft and $L = 350$ ft, using a maximum superelevation of 0.10 ft per ft.[23] Then $\phi = 10° 30'$; $T = 353$ ft; and $E = 22$ ft.

Each of the curves given is transitional throughout because $\phi = \Delta/2$.

Proportion Method.—Applying the proportion rule, with p equal to, say, one half ($\phi = \Delta/6$):

Curve 1.—$\Delta = 35°$; $\phi = \Delta/6 = 5°50'$; and $L = 250/2 = 125$ ft. Then $R = 614$ ft; $T = 256$ ft; and $E = 31$ ft. Using[24] $j = 0.16$ and superelevation $= 0.10$, allowable $v = 49$ miles per hr, an increase of 9 miles per hr over the conventional curve.

Curve 2.—$\Delta = 21°$; $\phi = \Delta/6 = 3°30'$; and $L = 350/2 = 175$ft. Then $R = 1{,}432$ ft; $T = 353$ ft; and $E = 25$ ft. Using[25] $j = 0.14$ and superelevation =

[23] "Transition Curves for Highways," by Joseph Barnett, U. S. Bureau of Public Roads, U. S. Govt. Printing Office, Washington, D. C., 1938, p. 55.

[24] *Ibid.*, p. 5.

[25] *Ibid.*, p. 6.

0.10, allowable $v = 72$ miles per hr, an increase of 12 miles per hr over the conventional curve.

To substantiate the fact that it is possible to permit speeds higher than those permitted by conventional theory, the following paragraph is quoted from an unpublished report by E. W. Allfather, dated July 20, 1940, describing the results of using an aeroplane bank indicator to determine the safe speeds to be painted on curve signs on the Blue Ridge Parkway in Virginia and North Carolina:

> "It was demonstrated that, for short double spiral curves, signs may safely show speeds fifteen miles per hour in excess of the theoretical speed obtained at the point of maximum curvature. However, it is doubtful if this rule could be applied to long double spirals as too much distance would be covered while traveling through degrees of curvature more nearly approaching the maximum degree obtained at the S.C.S. [spiral-curve-spiral]."

Fig. 3 proves the case against all-transitional curves so well that further proof seems unnecessary. However, for the record, Fig. 7 was plotted to show the theoretical values of j that would be obtained if a vehicle travels at 40

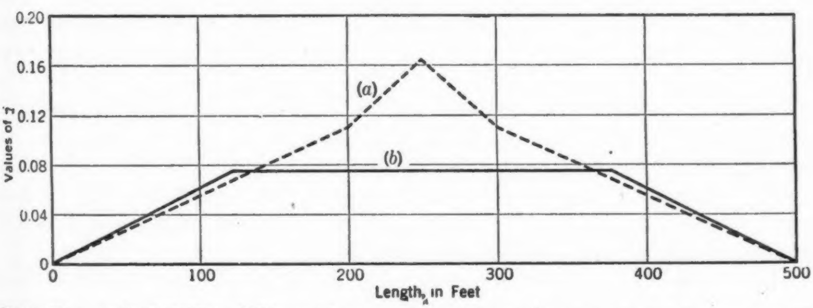

FIG. 7.—COMPARISON OF VALUES OF j AT $v = 40$ MILES PER HR AND 0.10 SUPERELEVATION FOR
(a) Double Spiral; $p = 1$, $R = 409$ ft
(b) Circular Curve with Spirals; $p = \frac{1}{2}$, $R = 614$ ft

miles per hr over the conventional double spiral, compared to those for the circular curve with spirals for which p equals one half. On the conventional double spiral, maximum superelevation was attained before the midpoint on the curve, as recommended.[26]

Although it is perhaps irrelevant to the present paper, the writer would like to know the values of j and Q which the observers described as "uncomfortable" and "slightly uncomfortable." It is presumed that correction was made for body roll in measuring j with the accelerometer.

In Section 1, the author noted that the formulas generally used for determining the length of required transition do not take superelevation into account. In 1936 the writer proposed a formula which allowed for the effect of superelevation[27] but recommended that it not be used because the lengths of transition

[26] "Transition Curves for Highways," by Joseph Barnett, U. S. Bureau of Public Roads, U. S. Govt. Printing Office, Washington, D. C., 1938, p. 9.

[27] *Transactions*, ASCE, Vol. 102, 1937, p. 1096.

computed with the formula were much too short for values of j near zero. This advice is in line with Mr. Leeming's observation that Q is unimportant. However, one should not overlook the fact that the minimum length of transition may be fixed by the distance required for changing from a crown section to a fully superelevated section, especially on multiple-lane roads.

In making a statistical analysis it is of paramount importance that cognizance be taken of all variables and special conditions. The author noted that his experiments were made principally with the British type of light car, at speeds seldom exceeding 50 miles per hr. Some of his tests were made on right-angle junctions and rotary intersections. It would be helpful if Mr. Leeming could describe the curves most frequently used in his tests. Very possibly, few, if any, of the curves he tested were comparable with the curves on the Pennsylvania Turnpike which have radii varying from 38,197 ft to 955 ft and an average length of about 1,400 ft.[21] It would be interesting to know what proportions of his tests were run on (*a*) curves without transitions, (*b*) curves with transitions, and (*c*) all-transitional curves. It seems likely that analyses of his tests by these three classes of curves would result in different constants for Eqs. 5 and 6.

To illustrate, assume two curves—one, a circular curve without transitions, and, the other, transitional throughout. The first curve is fairly common in the midwestern United States. The second curve has been used rarely if at all.

Circular Curve Without Transitions.—$\Delta = 90°$; $v = 40$ miles per hr; $R = 573$ ft; $L = 900$ ft; superelevation $= \frac{3}{4}$ in. per ft; and $j = 0.125$ (from Eq. 1). Solving Eq. 6, $p = 0.87$.

All-Transitional Curve.—$\Delta = 90°$; $v = 40$ miles per hr; $R = 573$ ft; $L = 1{,}800$ ft; superelevation $= \frac{3}{4}$ in. per ft; and $j = 0.125$. Solving Eq. 6, $p = 1.03$ (an impossible solution, as p cannot exceed 1).

In neither of the foregoing illustrations does p approximate values between two thirds and one half which the author states is the case for $j \geqq 0.1$. Perhaps the inference to be drawn is that the curves tested did not include many curves similar to those illustrated herein.

If driving tests to measure p should be made on the foregoing circular curve, the length of transition employed by the driver of the car could hardly exceed 300 ft because the offset required for inserting a spiral transition of that length is 6.5 ft. This means that, in order to stay on the road, the driver would probably make his transitions somewhat shorter than 300 ft. If he used 200 ft, the length of circular curve remaining would be 700 ft and the value of p would be $\frac{400}{400 + 700} = 0.36$. It is apparent, therefore, that the measured value of p would range from around 0.3 to 0.4 for a circular curve similar to the curve considered.

If driving tests should be made on the foregoing all-transitional curve, using the same line of reasoning, it appears that the value of p would range from about 0.8 to 1.0. This discussion should emphasize the desirability of qualifying statistical analyses such as those represented by Eqs. 5 and 6 by setting forth the limits of all variables.

In conclusion, it is believed that the author's proportion rule is applicable to all short curves, particularly those for which customary design methods result in values of p greater than two thirds. At other curves, especially those of great length, the value of p has little significance. Indeed, when the radius is greater than about 4,000 ft, there are no noticeable consequences if p is allowed to become zero, which is to say that the curve need not have transitions.

There is need for a comprehensive study to determine the length of transition required to satisfy the behavior of the average driver, but, until it can be determined for various radii and lengths of curves, and for a logical range of speeds, there seems to be insufficient justification for changing current practice.

Charles M. Noble,[28] M. ASCE.—In this refreshing paper the author questions the rather complacent attitude toward the subject of highway transition curve design.

Originally the cubic equation, generally utilized in the United States, was developed for railway practice and has been perfected by the railroads to suit actual operating equipment and speeds in the light of many years of practical experience. Safety and passenger comfort are the criteria. Fixed rails guarantee an accurate adherence to the designed path of the spiral so that the human equation in steering the vehicle is entirely absent. No real and searching investigation has been made to develop a transition curve suited entirely to highway vehicles and drivers.

As pointed out by the author and the writer,[29] the use of superelevation cancels out part of or all the centrifugal acceleration (depending on the speed of the vehicle), and this changes the fundamental assumptions in present spiral theory. Primarily, the transition curve enables the vehicle to attain superelevation safely within the lane path by avoiding the introduction of premature lateral forces, which are present if the superelevation is "run out" on the tangent ahead of the curve. If the spiral is long enough to attain this objective, there is some evidence to indicate that it will also be long enough to absorb safely the application of residual (after deducting the effect of superelevation) centrifugal forces.

These matters are clearly perceived by the author, and in the "Synopsis" he succinctly states:

> "* * * (1) that the theory is fallacious in its disregard of the superelevation, and (2) that the experimental evidence produced for it is questionable."

The writer wishes to support these statements and urges the implementation of an adequate, scientifically instrumented research program so that the real and practical facts relating to highway transition curve design may become known.

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[28] State Highway Engr., New Jersey State Highway Dept., Trenton, N. J.

[29] "Thoughts on Highway Design Research as Related to Safety of Vehicle Operation," by Charles M. Noble, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 17, 1937, p. 247.

In 1937 E. R. Haile, Jr.,[30] Assoc. M. ASCE, and T. T. Wiley,[31] Assoc. M. ASCE, proposed that spiral lengths be proportioned on the basis of rotational change in attaining the curve superelevation. The latter stated the railroads had determined that superelevation should not be introduced in track (4 ft $8\frac{1}{2}$-in. gage) at a rate greater tha $1\frac{1}{4}$ in. per sec of time at the design speed. This is equivalent to slightly more than 2% of cross slope, from which he concluded that:

> "Superelevation on a highway should not be introduced at a rate exceeding 0.02 ft per ft per sec, and the length of spiral on a curve with a superelevation of 1 in. per ft should approximate the distance traveled in 4 sec at the design speed."

It is worthy of note that this principle and the precise maximum value recommended by Mr. Wiley—namely, 2% cross gradient per sec of time—were utilized in designing the spiral lengths on the Pennsylvania Turnpike.

Apparently the author is not familiar with the high speed curve tests conducted on the Pennsylvania Turnpike, which were fully reported.[21] These tests were extended to the top speeds (105 miles per hr) of the stock car models used. A fifth wheel and electrical recording on flags spaced at measured intervals were utilized to obtain accurate speed measurements. Actual path measurements for both the front and rear of the car were taken and recorded. It is believed that a careful study of these test results by the author will be interesting and may suggest additional methods of approach to this problem.

The writer believes that in so far as possible all roadway features should be designed to conform with the requirements of both vehicle and driver,[32] and therefore wishes to support and emphasize the following quotation from the paper (under the heading, "3. The Rate of Turning the Steering Wheel"):

> "It is surely logical that the curve should be laid out to conform to the driver's practice; and, if he prefers, and habitually uses, some method of turning his wheel other than the constant rate expressed by constant Q, then the type of transition should be chosen to conform with his habits—even if this curve is not of spiral form."

This policy of matching road design with the characteristics of both driver and vehicle has received impetus in recent years and is now regarded as accepted procedure by American engineers.

As suggested by the author, it is entirely conceivable that driver-vehicle requirements may dictate another form of curve than that represented by the cubic equation. Experience in driving on the Pennsylvania Turnpike indicates that cubic equation spirals are faster than the simple curve they connect and may have a tendency to lure the driver into the curve too fast. This is not serious, but it indicates that the cubic equation may not be the ideal sought.

There is some indication that it may be desirable to treat sharp curves and flat curves differently. It is believed that there has been confusion in dealing with the subject of transition curves as between sharp and flat curves.

[30] *Transactions*, ASCE, Vol. 102, 1937, p. 1097.

[31] *Ibid.*, p. 1105.

[32] "Engineering Design of Superhighways," by Charles M. Noble, *Proceedings*, Am. Road Builders' Assn., 1941, p. 183.

Some proponents advocate making the curve transitional throughout, and even Mr. Leeming proposes that the transitions should be a fixed proportion of the entire curve. To most practical American highway engineers it appears preposterous to transform a simple curve 3,000 ft or 4,000 ft long into transitional curves throughout, or to have a fixed proportion of their length transitional. The tendency of drivers to "cut corners" on entering a curve should also be considered as one of the factors in the problem.

To develop further the sharp-flat curve idea, it may be illustrative to divide curves arbitrarily into two groups: Those sharper than 1,000-ft radius and those flatter than 1,000 ft. There is a possibility that the cubic equation may be suited to curves sharper than 1,000-ft radius, whereas another form of transition may be required for easier curves. There is also some evidence that curves flatter than a 3,500-ft radius do not require spirals at all for design speeds up to 70 miles per hr.

For example, assume a curve with 3,000-ft radius, a superelevation of 0.03 ft per ft, and a design speed of 70 miles per hr. (On modern American main trunk highways a curve of this radius may vary from 500 ft to 5,000 ft in length.) If the rotational theory for determining spiral length is applied, a 154-ft spiral is required. (For simplicity, it is assumed that the roadway surface is level at the beginning of the transition.) This will result in a "throw" of 0.34± ft

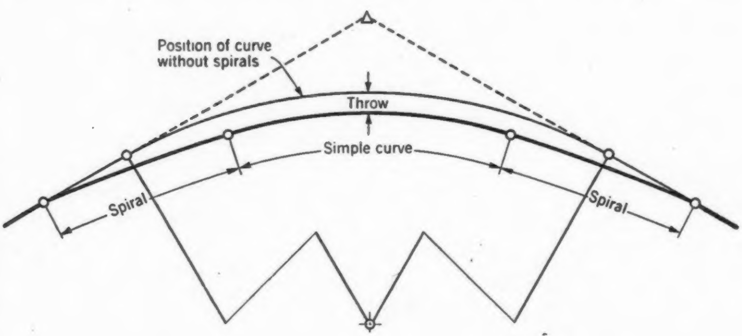

FIG. 8

(Fig. 8). This offset or throw is so small that it does not appear to meet the requirements of drivers who cut corners on entering curves. If Mr. Leeming's proportional rule is applied, and assuming that the 3,000-ft radius curve is 1,260 ft long, then a 420-ft spiral is required, which will result in a throw of 2.5± ft. This throw or offset may more nearly meet the corner cutting tendency of the driver, but if a cubic equation is utilized much of its length will be of such slight curvature that it is meaningless in practical driving. To be specific, a driver will traverse 211 ft of the spiral before the curve sharpens to a 6,000-ft radius, and 360 ft, before reaching a 3,500-ft radius. Thus, it appears that the throw may meet the requirements of the driver, but that the curve is too flat on the end and should be shortened 210 ft, indicating a transition curve length of 210 ft. However, such a curve must have different characteristics

from a cubic equation in order to obtain the desired throw within a length of 210 ft.

On the other extreme, assume a curve with 500-ft radius, a superelevation of 0.10 ft per ft, and a design speed of 43 miles per hr. If the rotational theory is applied, a 315-ft spiral is required, which will yield a throw of 8.24± ft. This offset may be entirely sufficient to meet the corner cutting tendency of the driver, whereas the offset of 0.34 ft for the cubic equation spiral on a 3,000-ft radius curve appears insufficient. Furthermore, it is necessary for the driver to traverse only 45 ft of the 315-ft spiral before reaching a radius of 3,500 ft. If the author's proportional rule is applied, assuming the curve is 700 ft long, a 230-ft spiral is required, which will result in a throw of 4.4± ft. It is believed that this transition is too short for a 500-ft radius curve.

In considering the operating possibilities on the 3,000-ft and 500-ft curves, it is interesting to note that increasing the speed on the 3,000-ft curve from 70 miles per hr to 80 miles per hr will increase the unbalanced centrifugal ratio, f, to only 0.113, whereas increasing the speed on the 500-ft radius curve from 43 miles per hr to 53 miles per hr will increase f to 0.275. Therefore, the longer transition will give drivers who enter the sharp curve too fast an opportunity to slow down. In addition to the design of the transition curve and its length, proper methods of warping out the tangent roadway crown must be devised if the entire design is to be acceptable to the motorist.[33] Consideration must also be given to rate of longitudinal grade change along the pavement edges.

It is hoped that this paper will stimulate additional investigation involving scientifically instrumented road tests so that this phase of highway design may be properly established. The writer does not believe that the proportional theory advocated by the author will meet driver-vehicle requirements, inasmuch as a transition of different length and throw would be utilized for the same radius curve whenever it varied in length. Until new forms are proved superior in meeting driver-vehicle requirements, use of the cubic equation will undoubtedly continue in the United States. The author is to be congratulated for the presentation of material on an important subject, and it is hoped that he will continue his study and investigation.

[33] "Developments in Curve Design, Speed and Sight Distance," by Charles M. Noble, *Roads and Streets*, January, 1942, p. 25.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RIVER INFILTRATION AS A SOURCE OF GROUND WATER SUPPLY

Discussion

BY RAPHAEL G. KAZMANN

RAPHAEL G. KAZMANN,[4] Assoc. M. ASCE.—At the time the water supply of the Indiana Ordnance Works was conceived, information was available concerning only one similarly planned major infiltrated water supply in the United States—the Des Moines, Iowa, filtration gallery. Consequently, Mr. Youngquist's discussion is, in a sense, a measure of the advance in ground water engineering during the seven years since 1941, and in particular during the five years since 1943. In the course of developing from a qualitative, descriptive science, ground water hydrology has reached the stage of as much a quantitative science as surface water hydrology, and probably is more susceptible of yielding reliable quantitative answers than is applied mechanics.

In the field of water supply, ground water hydrology recognizes, and has reliable quantitative answers for, the following categories of problems when encountered in aquifers composed of unconsolidated materials:

1. An underground water supply based on infiltration—

(*a*) From rainfall on the immediate area where the wells are located;
(*b*) From rainfall on a distant area where the aquifer outcrops; and
(*c*) From rainfall and from a river or lake touching a distant outcrop area.

2. An underground water supply based on infiltration from a near-by source such as—

(*a*) A permanent river or lake that can readily supply all desired quantities of water if the aquifer will transmit them (for example, the Mississippi, Missouri, and Ohio rivers and their larger tributaries or, say, Lake Michigan);

NOTE.—This paper by Raphael G. Kazmann was published in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows, March, 1948, by C. V. Youngquist.

[4] Chf. Hydrologic Engr., Ranney Method Water Supplies, Inc., Columbus, Ohio.

(*b*) An intermittent stream crossing deep buried valleys, such as Nimishillen Creek in the Canton (Ohio) area;

(*c*) A "flashy" stream to furnish a variable-quantity infiltration supply which will supplement existing surface water sources during dry weather and furnish large supplies during floods, when the turbidity of surface water forms a major treatment problem for water works operators;

(*d*) A permanent or intermittent stream with sufficient subsurface storage to furnish water with a small, predetermined temperature fluctuation, as compared to the large temperature variation of the surface water which is supplying the aquifer; and

(*e*) A permanent or intermittent stream with sufficient subsurface storage to furnish water whose mineral quality will fluctuate only slightly as compared to the variations in quality encountered in the surface source, in order to minimize water treatment problems.

These problems are not listed in order of importance or magnitude. The Indiana Ordnance Works is classified in category 2(*a*) as is the supply of the City of Manitowoc, Wis. The use of such a stream as that listed under category 2(*b*) as a source necessitates knowledge of flow-duration curves, the dimensions and contour of the underground reservoir, and the storage coefficient of the reservoir to determine the yield, as Mr. Youngquist has shown. In addition, the physical characteristics of the water already in storage should be determined, as well as the approximate quantity of areal rainfall reaching the underground reservoir, although this is usually a relatively small quantity. The recently completed stream infiltration supplies of the cities of Anderson, Ind., and Canton are classified in this category. In category 2(*d*) both the quantity and the approximate temperature variations of the water can be predicted.

Of course, there are other specialized water supply problems for which reliable quantitative answers can be obtained, but the bulk of all underground water supply problems are included in categories 1 and 2.

The reverse side of the water supply problem has not been touched, of course, in either the original paper or the discussion—that is, the problem of subsurface drainage and the removal of unwanted water from specific areas. The same quantitative ground water procedures are applicable to dewatering problems as to problems of underground water supply.

One paragraph of Mr. Youngquist's discussion is of special interest to hydrologic engineers because it summarizes, perhaps too briefly, the use of underground storage in an infiltration system. This paragraph compares the magnitude of withdrawals from an aquifer with withdrawals from a surface reservoir.

The function of the natural, underground reservoir is the same as that of a surface reservoir—to supply peak loads and to satisfy a predetermined minimum water demand during periods of low flow in the stream. The rate of infiltration to the aquifer (the equivalent of flow rate into a surface reservoir) is dependent on (1) the quantity of water flowing in the stream at any time; (2) the velocity of the stream; and (3) the stage of the stream relative to the general water level in the aquifer.

With the increase of each of these major factors, the rate of infiltration increases. If the subsurface storage is more or less depleted, as it will be at the end of a dry summer, the rate of infiltration during floods will be a maximum and will undoubtedly exceed the indicated 5 mgd or 6 mgd per acre of stream bed proposed by Mr. Youngquist. The bulk of recharge to an underground reservoir occurs during floods for precisely the same reason that surface water reservoirs are filled by floods: Most of the water is there at that time.

Summary.—Underground reservoirs do not remove productive land from cultivation and, equally important, their storage capacity never decreases because they do not silt up. The planners of new water supplies should not overlook the available underground water in an area. Such supplies can be relied on to furnish perennial, and predetermined, quantities of water.

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DISCUSSIONS

APPLICATION OF GEOLOGY TO TUNNELING PROBLEMS

Discussion

BY THOMAS W. FLUHR, PORTLAND P. FOX, HYDE FORBES, AND ROGER RHOADES, A. B. REEVES, AND W. H. IRWIN

THOMAS W. FLUHR,[14] Assoc. M. ASCE.—In spite of the fact that the works of the engineer are founded on, or pass through, the materials of the earth's crust, papers on geology seldom appear in engineering publications, and too few engiineers have sufficient acquaintance with the subsoil and bedrock that concern their projects so vitally. The subsoil and bedrock vary to an extreme degree in quality and condition. The engineer designs his steel and concrete structures with the indicated degree of precision and then founds them on natural materials, the quality and condition of which are known only approximately and, unfortunately, in some cases not at all. This paper therefore serves an extremely useful purpose in that it delineates some of the geological features encountered in tunneling in such a manner as to be easily comprehended by an engineer, and in such form that he may make practical use of them in planning and construction. Since the kinds of rock and soil and their characteristics vary considerably in different locations, opinions may differ on the emphasis to be placed on various geologic features.

The engineering geologist is most valuable to the engineer prior to the beginning of construction. Reconnaissance of a proposed tunnel line often is sufficient to discover the critical points of the line. Geologic mapping and structural studies will permit the geologist to predict subsurface conditions and anticipate difficulties with greater assurance. Programs of test borings, test pits, or other methods of exploration can be devised and their progress guided by geological considerations.

When the subsurface features have been explored, the engineer and the geologist, working together, may find it possible to shift a tunnel line to advantage. For example, one section of the Delaware aqueduct tunnel line was shifted laterally to avoid passing through the intersection of two fault zones.

NOTE.—This paper by Ernest E. Wahlstrom appeared in October, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1948, by Berlen C. Moneymaker, and Robert S. Mayo; and March, 1948, by F. A. Nickell, R. H. Keays, and Jacob Feld.

[14] Senior Geologist, Board of Water Supply, City of New York, Downsville, N. Y.

In another case, part of the tunnel was placed much deeper than planned, and thus decayed rock present at the original level was successfully avoided. However, such shifts in position can seldom be made, because they usually involve increasing the tunnel length which results in more expense than would crossing bad ground.

The engineering geologist, by predicting the location, quality, and extent of bad tunneling ground, makes it possible to plan for such eventualities when cost estimates are prepared. His reports are frequently of great service to contractors bidding on proposed work. Geologic forecasts should not be included in specifications. On the other hand, the proper manner of presentation of geologic data is of advantage both to the engineer and to his client, and to the contractor, and the withholding of such geologic data as are available may be construed by a court as evidence of bad faith. One practical aspect of making geological data available to the contractor is the possible elimination of future claims.

Once construction has begun, the geologist can furnish advisory service to the engineer regarding tunneling conditions and can record the geology of the tunnel for future reference, but unless wet or caving ground or serious fault zones are encountered, he can do little to change the course of events.

It would appear that in the paper too much weight has been placed on "flow layering" of igneous rocks. The directions and intensity of the jointing are of much greater importance. Not everyone will agree that a greater amount of overbreak occurs in tunnels driven normal to the foliation or bedding. The overbreak depends to a great extent on the jointing, as well as on the manner of drilling and shooting. Similarly, the problem of driving tunnels so that the layers dip away from the heading is of little practical importance, and can be taken care of by a skilful heading boss.

One point emphasized by Mr. Wahlstrom is the effect of ground water on tunneling. In the eastern section of the United States, the writer has observed that less difficulty is usually encountered with the badly decayed central crush zones of faults, than with the jointed and water-bearing zones on each side of the decayed zone. Control of water is of first importance both in hard rock and soft ground tunneling. In general, when an inflow of water is encountered in a tunnel, the average engineer tries to shut it off by grouting, usually without success. Most inflows diminish or disappear if allowed to flow; if shut off, the hydrostatic pressure builds up to that corresponding to the ground-water level, and eventually the water breaks through elsewhere.

When a water-bearing fault, sometimes accompanied by a run-in of soft material is encountered, the contractor, anxious for progress, often feels that he has to take some immediate action, and it is difficult, if not impossible, for the geologist to persuade him to wait until the water has drained off and the soft ground has become stabilized. One must recognize that in underground, as well as in living bodies, nature tries to heal her wounds.

Modification of the cross section of a tunnel to conform with the geologic structure is theoretically desirable, as Mr. Wahlstrom points out. However, tunnel driving today is a highly systematized procedure, and any change in section is expensive in terms of time and progress. Moreover, since a large

proportion of tunnels are concrete lined, changing the section would necessitate the use of special and expensive forms for lining, or else the use of excessive amounts of concrete to fill the additional excavation.

One problem, not mentioned by the author, is that of popping or bursting rock, which often necessitates the use of roof support.

Mr. Wahlstrom is to be commended for having presented his material in a form which is simple and direct, as well as technically correct, thus enhancing its value to those engaged in tunneling work.

PORTLAND P. FOX,[15] Assoc. M. ASCE.—Many civil engineers will find this paper interesting and instructive, but experienced engineering geologists will feel that it is somewhat academic. The civil engineer will have difficulty in interpreting many of the geologic structures from outcrops, drill cores, and other explorations. In many regions there are no rock outcrops from which a geologist or engineer can determine the character of the joints and faults and only information obtained from core drill holes will be practical or possible.

If a core drill hole was drilled through faults or fault zones as represented in Fig. 4, few geologists or engineers would be able to recognize the fault (much less appraise its significance) unless a nearly perfect core recovery was obtained, which is rare in such zones. Such fault zones could be represented by a few feet of core loss and pass completely unsuspected until encountered in the tunnel. Geologists may easily classify the faults after they are encountered in the tunnel, but it is difficult to do it before.

The author states (under the heading, "Physical Characteristics of Consolidated Rocks") in discussing the relative merits of the geologic structure represented in Figs. 2(*a*) and 2(*b*) that "the latter condition should be avoided if possible." The writer has found, in tunneling in the Brazilian Archean complex, that the structural conditions represented in Fig. 2(*a*) will cause much more overbreakage and many more tunneling problems than those shown in Fig. 2(*b*) if the dip joints are well developed and if there is slight weathering along them. This is especially true if the dip joints strike at an acute angle with the tunnel.

It is not practical in the planning and construction of most tunnels for civil engineering works to aline the tunnel to suit best the geologic structures. The writer assisted in the planning and exploration of the 7,100-ft tunnel for the relocation of the Chicago, Burlington and Quincy Railroad Company in the Wind River Canyon in Wyoming for the construction of Boysen Dam. A detailed geologic study was made of the area and it was immediately realized that several large fault zones would be encountered by the tunnel, but even so the tunnel was more feasible than the alternates.

The author states (under the heading, "The Tunnel Cross Section"):

> "Much time and effort can be saved by arching the roof in such a manner as to utilize what strength there is in the caving rock until supporting structures can be set in place."

Generally speaking, this would not be practical because it would involve a different tunnel section in caving and noncaving ground, and, if the tunnel had

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to have a permanent concrete lining, the forming of such an arch would be objectionable.

The writer has usually found that coarse-grained rocks alter more rapidly than fine-grained rocks, contrary to the author's statement (under the heading, "Altered Rocks and Swelling Ground"). For instance, in the Wind River Canyon in Wyoming the fine-grained granite gneisses underlying the Flathead sandstone are rarely altered, but the coarse-grained pink granites are usually weathered several feet below the sandstone. Pegmatites are usually more deeply weathered in Brazil than are the adjoining finer-grained wall rocks.

HYDE FORBES,[16] M. ASCE.—This well-considered outline of the application of geology to tunneling problems is directed more to the student geologist or to the geologist employed in such work than it is to the engineer responsible for design and construction. It is to be hoped, therefore, that discussion of this comprehensive paper will yield field experiences so that the engineer, although possibly not fully able to recognize the rock conditions with which he is confronted, will know when to seek the advice of a geologist who is able to do so, and learn how to appreciate the importance of obtaining a geologic interpretation.

It is true that the preliminary investigation and the actual construction of tunnels involve a most direct and detailed application of the science of geology within the field of activity of the engineering geologist. Unfortunately, however, this is usually limited to the preliminary investigation of ground conditions by interpretation of diamond drill cores, or as revealed through shaft and other type exploration. From these all too limited "peeks" into the subsurface the geologist is supposed to give the engineer a complete geologic cross section of the formations to be bored, and geologic interpretation or determination of the character of the ground as to the occurrence or nonoccurrence of fractured, seamy, or noncoherent rock; water; and gases from point to point. In addition, he must solve the practical problem of the load to be carried by the tunnel support or lining, and the magnitude and distribution of ground or water pressures on which to predicate plans and specifications for the tunnel.

Excavation for tunnels through sound, hard rock has not presented any exceptional features. The section generally can be taken out with a center top heading and one or more benches (depending on the height and width of the section heading) with the spacing and number of drill holes depending on the hardness of the rock. The engineer achieves a confidence in his ability to judge and handle rock. It is, therefore, well to place emphasis on the necessity for "full coverage" through the employment of a geologist throughout the life of a job. In nearly every length of tunnel some modifications of the rock due to geologic processes are to be expected; these may be enumerated as follows:

(1) Heavy ground, wherein hard but "blocky" rock exists with the blocks of hard rock separated by bedding planes, planes of schistocity, and fractures or joints opened either through geologic processes in the zone of surface weather-

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ing or through faulting and intense folding, or through blasting in the tunnel itself, subject to fall if not supported;

(2) Seamy ground, wherein the rock walls of fracture or other planes are found separated by varying thicknesses of the products of rock decomposition, produced by the action of water and in which water is encountered;

(3) Swelling ground of fault zones and that due to chemical or physical action when the bore exposes the rock to contact with air and moisture and when it removes retention and support; and

(4) Flowing ground, which includes plastic clay of high moisture content and saturated sand.

Designers and constructors can cope with these types of ground by methods which have been developed through experience. It is the function of the engineering geologist to interpret the field conditions and to provide working data of value for design and construction in the event that these forms of rock weakness, disintegration, and decomposition can be anticipated or found to occur after construction starts.

In discussing the physical characteristics of consolidated rock the author presents a picture too involved with geologic terminology to mean much to the engineer. The writer's contact with the profession has shown him that the civil engineer studied the prescribed geology courses in college, and that the professor generally maintained the position that the science is an end in itself, rather than that it is of extreme value to the engineer as a means to an end; and so the student understood and retained little of the "book thrown at him." The same thing applies to a general discussion of rock structure. Generalities on the subject are of little value and it is difficult to state which particulars have wide application or avoid being misleading if not correctly identified. The writer has found that when rock cores show slickensides when broken, or swell when they are removed from the core barrel, it is an indication that actively swelling ground will be encountered in the tunnel bore. Fractures of any kind that carry gouge seams or oxidized products with more than a minimum thickness require strong support, and probably will carry water when pierced by the bore. The clayey gouge of fault zones and moist kaolinized (clayey) walls of faults and fractures act as viscous fluids under the load pressures produced by overburden. These materials are seldom recovered in the core.

Actively swelling ground due to chemical causes depends on the combination of minerals in the rock with the chemically active substances in the air. Air slacking is the general term used to cover oxidation, carbonization, and hydration. Much can be predetermined in that regard by tests on rock cores. The difference between the density of a rock upon its removal from the core barrel, and after it has stood several days exposed to the oxygen and carbon dioxide of the air, indicates the volume of swell taking place. The action of some rocks upon submergence in water is an indication of the amount of hydration to which they will be subject upon contact with moisture. Swelling ground results from the integration of minute slippages between extremely small fragments or crystals in rock material. This may be due to hydration, as

in serpentinization, resulting in a slickensided rock. In driving a tunnel through serpentine under Parker Avenue (San Francisco, Calif.) blasting was necessary to remove the massive rock, but within a short time the further hydration of the rock produced clay which could be scraped from the walls and roof to the extent that the workers classed all the material as clay.

Swelling results from pressure (essentially hydrostatic) in fault zones characterized by crushed and ground-up rock (gouge). The outlet tunnels from the San Andreas reservoir of the San Francisco water department are bored through the San Andreas fault zone, the most extensive and active rift in California. The material making up the zone is gouge and breccia, the latter consisting of blocks of unground rock (called boulders by the miners), which are segments of the original rock that have slid in the finely ground gouge in adjustment to the pressures and have suffered only compression. The gouge is made up of a multitude of minute dry particles, between which slippage occurred as soon as the bore pierced it. The initial swell amounted to as much as 8 in. and the working material developed slickensided planes in the wall and at the contact between gouge and breccia. Swell continued for 36 hours as the pressure acting on the gouge at tunnel level was gradually relieved by the shearing stresses or "arching" developed in the gouge, until a new condition of pressure equilibrium was achieved in the mass around the bore. The material was cut back to line when the swelling ceased and the load was successfully carried by the timbers. When the tunnel was completed and approved for lining, blasting in a near-by quarry regenerated movement as the stress-strain equilibrium was upset by shock waves. The result was a new "working" of the material; new slickensided planes developed, and breccia boulders fell from the roof as shown in Fig. 7.

Rock falls and excessive overbreak due to blocky ground, characteristic of the weathered zone of all formations as well as where folding and faulting has been geologically recent and intense, are well described by the author (under the heading, "Folds, Joints, and Faults"). This can be foretold through a surface geological survey and mapping. A case in point was the Broadway Low Level tunnel in Oakland, Calif., approaching and through the Wild Cat fault zone. The rock formation exposed in road cuts was sheared to the extent that its original structure was hardly recognizable. This led to the conclusion that overbreak in the tunnel would be excessive and rock falls would be a common occurrence.

In this regard, as well as in geologic mapping during the construction of a tunnel, it is well to place emphasis on the employment of a geologist throughout the life of the job. The writer wishes to underscore the author's statement (under the heading, "The Functions of Geologist"):

> "The importance of a detailed geologic map of the tunnel cannot be overemphasized. This map not only should show the geologic structures and rocks encountered, but it also should serve as a lasting record to indicate the location of bad sections, which may require attention many years after the tunnel is completed."

An example of neglecting to provide such a map is found in the Coast Range tunnels of the Hetch Hetchy aqueduct system of San Francisco; nor is the

writer aware of such a record for other important tunnels bored in California in the past twenty years.

As to the location of a tunnel, the geologist is rightly limited to the selection of the best of alternate locations which will serve the proposed engineering requirements, and frequently there is no alternative in relation to the purpose of

FIG. 7.—ROOF AND WALL OF SAN ANDREAS OUTLET TUNNEL AFTER QUARRY BLASTING

the tunnel. It is seldom possible to change the direction of the tunnel to meet changes in rock structure as the author suggests, but the author briefly mentions (under the heading, "Folds, Joints, and Faults") that the geologist can advise in connection with the important feature of rock structure in relation to blasting. Forty years ago during precollege and college vacations the writer worked

in California gold mines. Then, much was learned of tunneling from the experienced, practical miner, but the knowledge of rock and the application made of that knowledge, which was most interesting and impressive to this student of geology, was found in studying the rock to determine its "cleavage." The location, number, depth, and powder charge of drill holes were dictated by this cleavage, and the foot or hanging wall of waste rock would be cleanly stripped from the gold bearing quartz vein without fracturing or damaging the vein in any way. After the tunnel was mucked out, the vein would be drilled and shot down on canvas without breaking into or carrying down waste rock.

The writer has found it difficult in his professional work to impress upon engineers the idea that most rock has incipient cleavage, because of compressive stresses to which it has been subjected in crustal movement, or swell of adjacent igneous masses in geochemical change. In addition, bedding planes of sedimentary formations, planes of schistocity, or other structural features of rock character contribute prominent lines along which rock cleaves. These cleavages should be given consideration and should be as a guide for using powder to break the rock. In considering the dam sites for the State Wide Plan of California (later the Central Valley Project) from 1928 to 1930, the writer endeavored to present specifications covering stripping of the sites. The reaction is best recorded in the remark of one consulting engineer that a contractor will not strip a dam site with a nut pick.

In connection with the sites selected in 1934 for debris dams on the Yuba, Bear, and American rivers to be constructed by the United States Army Engineers in California, the writer presented a definite stripping pattern on maps of the sites, meeting the cleavage of the rock and utilizing the banding due to compressive stress so that a minimum of rock had to be removed to provide a sound, unyielding, strong, watertight foundation and abutment. This was not understood, nor was its significance appreciated by the designing engineer, and it was unwelcome as an unwarranted trespass by the geologist into matters within the prerogative of the designer.

The ruination of good dam sites and the damage to tunnels observed by the writer have been largely due to "poor rock" caused by improper blasting. Much rock in California does not require blasting with the modern excavation equipment now available. In a recent rock excavation project in a built-up area in San Francisco the writer prescribed slopes to meet the rock conditions, based on boring and sampling, and specified that no blasting was required. It was believed by the engineers that bids could not be obtained on that basis, and a compromise was reached permitting the drilling of holes no deeper than 3 ft and loading with not more than 2 lb of powder. The contract is nearing completion without resorting to this limited use of blasting, except in the case of two large rock masses, one rolling down the slope, and another projecting above finished grade.

The observation of the diamond drill operations and the study of (and tests on) core recovered in connection with the prospective Broadway tunnel in San Francisco led to the recommendation that a tunnel be driven without blasting. The "chittering" of the drill so shattered the rock cores that less than 25% core recovery was effected. The recovered cores had a compressive strength ranging

from 530 lb to 1,750 lb per sq in. when tested unconfined, but they shattered to small fragments under a percussive force of from only 5 ft-lb to 9 ft-lb. It was clearly shown that cleavages were so numerous that no piece of rock larger than a 1-in. cube would resist a sharp hammer blow. An important function of the geologist is to determine the presence of lines of structural weakness in rock and the methods of treating them in the process of tunneling, as well as in other excavation. It will take considerable persuasion to convince the engineering profession, if the writer's experience is any criterion.

The author states that the prediction of ground-water conditions in tunnels is one of the most baffling problems confronting the geologist. Experience will soon teach the observer that much can be predetermined in relation to the source, occurrence, and effect of water in tunneling. Tunnels driven under lakes, rivers, or other bodies of surface water can be expected to tap considerable volume of flow. Those driven under regions in which wells or springs are used by surface landowners for water supply purposes are found to dry up those sources, and tunnels driven into some formations release highly mineralized stored ground water that contaminates streams into which the tunnel drainage flows. Surface mapping of all water sources in a considerable area on both sides of the tunnel line should be carried out, and the probable water table beneath the area should be plotted. The relation of water bodies or courses to faults or other geologic structural features should be noted on the map. During exploration, drilling notes made of the depths at which water is lost or rises in the drill hole are of value, and measurements of water level in the hole at the end of each day's run and before the next run, to determine the loss or gain, will be found helpful.

Some water troubles occur where fissures filled with water are encountered. These fissures may range in width from hair thickness to several inches, and the water pressures may be the full hydrostatic head to the surface at the highest point of the water table. The Santa Barbara tunnel in California was bored through almost vertical dipping, sedimentary rock beds and water was encountered along bedding planes that had been opened by blast shock; the water pressures were sufficient to cause the rock to "pop" dangerously (fatal to several workmen) before the water seam was reached by the bore.

The city tunnel of Santa Barbara was driven a total of 18,000 ft through sandstone and shale formations characteristic of the Coast Range of California. Because the rock was not of a porous nature, it would have yielded little water except that, as the work progressed, the incipient, closed fractures (due to distortion suffered by the formation in crustal movement) and bedding planes were made crevices through blasting. These opened to the ground surface and filled with water from rainfall and soil water percolation. The tunnel portal is at El. 1500; the range rises to El. 5000. The writer made a survey of the springs that had been dried up by drainage through the tunnel in 1915 and found that the drained area extended approximately 3 miles east and west of the tunnel section at the crest of the range, in the form of an inverted cone with the tunnel at the apex. The tunnel was lined with concrete, but, because of water pressures, weep holes were installed, and the tunnel developed a considerable inflow—ranging up to 2,500 gal per min with the amount of seasonal rainfall. The flow

from the tunnel, since used for city water supply, was fairly constant in that the sections nearer the portals discharged the greatest amount soon after (and during) the winter rains, whereas the flow from sections at greater depth below ground surface increased as that at the portal sections diminished.

The same experience was encountered in the boring of the Lafayette tunnel of the East Bay Municipal Water District of Oakland, through a geologic formation of the same character. There, wells of dairies located on the ridge were deprived of water, whereas formerly the wells, more than 100 ft in depth, had a water level always standing between 10 ft and 20 ft from the surface—about 800 ft above sea level. The tunnel at El. 370 passed about 1,500 ft south of one well and down the dip of the strata from it, and the blasting within the bore opened the bedding planes beyond the well, as indicated by the drainage of water from the formation.

Similar conditions were brought about in driving the Mono tunnel of the Los Angeles (Calif.) Aqueduct in 1935. The formation through which the tunnel was driven, and from which springs issued at the surface, was volcanic in origin, varying greatly in mineral composition, and occurring as beds of lava, tuff, and agglomerate. It was dislocated by faulting, and the members of the volcanic series had a decided dip from their original horizontal position so that the tunnel penetrated them in succession. The bore, 9 ft by 10 ft, was driven approximately 300 ft below the spring area. Blasting in the tunnel opened the fractures and contact planes in the various formations and drained the water into the tunnel at rates in excess of 100 gal per min, drying up the spring area.

The effect of opening planes of schistocity by blasting was found when a state highway was carried over a tunnel of the Western Pacific Railroad near Keddie, Calif. The tunnel was bored and lined and had been in use several years. The blasting of the rock for the highway cut over the tunnel opened the formation along the planes so that segments of the rock were loosened and slid along the self-lubricating micaceous schist, buckling the tunnel lining. The character of the rock and the dip of the planes directly over the tunnel are shown in Fig. 8. In this instance, lubrication of the sliding plane through moisture was not required to facilitate movement. In tunnels, however, most rock slips occur when clayey or talclike alteration products resulting from the action of ground water on the rock and filling joints, bedding, or fracture planes, or those making up the gouge of fault seams or walls of joints, are softened when tunneling operations allow water to move down the slightly opened crevice.

The author suggests the alteration of the tunnel section to meet the caving of the walls or roof of a tunnel. The walls and roof of the tunnel become unstable due to conditions brought about by the tunnel, after it has been bored through the troublesome formation, and little can be predetermined which would indicate the necessity for a change in cross section, even if the engineering requirements should allow such change. Moist clayey decomposition products form a consequential part of a thoroughly fractured rock mass through which water has circulated, and the mass will be incapable of withstanding even moderate stresses when restraint is removed by the bore. Faults that carry gouge seams more than a few inches thick in humid regions are usually found

to have carried water, even if no water is found in the preliminary drill hole. The rock hydrates to the extent of taking up the water trapped in the fractures, and the presence of water is not revealed. Strong timbers will be required to meet the imposed forces, similar to hydrostatic pressure, as the clay gouge and kaolinized walls of a fault or fracture will act as viscous fluids under the pressures at tunnel level regardless of the cross section.

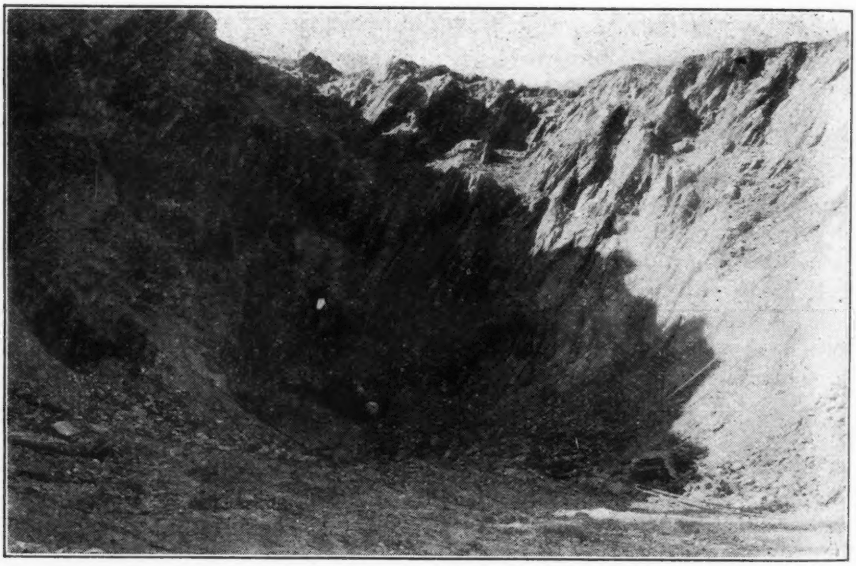

FIG. 8.—FACE OF HIGHWAY CUT EXCAVATION OVER TUNNEL

The author states (under the heading, "The Functions of the Geologist"), that in relation to cooperation between the engineer and the geologist:

> "* * * this situation is gradually becoming less serious and the engineer and the geologist are reaching an amicable understanding based on mutual respect and profitable cooperation."

This is true among the salaried employees or civil service employees in large corporations and governmental agencies. Nevertheless, the "awareness" of the importance of geological advice from a consultant of broad experience is not so well developed in the administrative head or the engineer in responsible charge. The writer's experience is that he has been employed after a tunnel is constructed, rather than before and during construction, on a ratio of five to one—usually to investigate: (1) Damage done by blasting in the tunnel causing incipient jointing to be opened as fractures reaching to ground surface, along which the ground settles and thus affects the foundations of structures, or along which water drains to the tunnel depriving overlying lands of their water supply; (2) damage done to tunnels due to blasting in their vicinity; (3) contamination of streams through tunnel drainage; and (4) contractor's claims for additional money, based upon the contention that the engineering design

was faulty when applied to the particular ground conditions. These investigations have been most instructive, but have also afforded the opportunity to observe that, considering the money involved in the project, only one of several obvious recommendations was needed, had an experienced engineering geologist been employed, to save the cost of the geologist's fee several times over.

The engineer in responsible charge of tunnel projects should insist on having the assistance of an engineering geologist. In turn, he should provide the geologist with such preliminary exploration as he recommends and believes necessary to present the occurrence or nonoccurrence of consequential fault zones, structural weaknesses in the form of extensively jointed, fractured, disintegrated, or altered (decomposed) rock, and their locations. Sufficient funds should be supplied also for tests on material occuring at tunnel level to determine what degree of decrepitation the material will suffer on exposure to air or water. Provision should also be made to take sufficient rock samples or cores representative of the rock profile above the tunnel, from which the pressures at the tunnel arch can be determined, as well as to ascertain such properties of the rock as elastic expansion upon release from restraint, swell, compressive strength, and resistance to percussive force; and such observations and tests should be progressively carried on during the driving of the tunnel as a basis for current recommendations and final record. All these are well worthwhile, to the end that safe and economical tunnel design and construction will be better founded on fact.

ROGER RHOADES,[17] ESQ., A. B. REEVES,[18] M. ASCE, AND W. H. IRWIN,[19] ESQ.—It is now accepted as a truism that geology can contribute helpfully to civil engineering. The author has restated some aspects of the geologist's role in tunneling, has described or mentioned several basic geologic principles which may be applied profitably to tunneling operations, and has alluded to a few specific examples of such application.

The object of this discussion is not to disagree with either the general thesis or the specific content of the paper; it is written soundly, from the geologic standpoint, and, if tunnel engineers uninitiated in geologic matters obtain from it a more sympathetic understanding of the medium—the rocks and soil—with which they work, the author's objective has presumably been achieved, and a useful purpose has been served. However, a large and complex subject is involved and the author, constrained to brevity, has dealt only with the basic fundamentals. The purpose of this discussion is to point out the difficulty of compressing a subject so large into a presentation so brief, and to warn of the danger that the uninitiated—among both engineers and geologists—may fail to comprehend the full extent of the potential contribution of geology to tunneling and the degree to which cooperative geological-engineering studies are being applied currently to tunneling operations.

The extensive and increasing participation of geologists in civil engineering operations in general is now a matter of common knowledge. In the field of

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tunneling this geologist-engineer partnership should be closer than in any other kind of civil engineering because the problems met by the tunnel engineer arise almost wholly from the rocks, soil, and ground water which comprise the geologist's special field of professional interest. The engineer requires geological information for the most effective and economical planning and construction of tunnels; the geologist is challenged to procure and supply the engineer with the information he needs, in terms that are as precise and quantitative as possible—not in the form of generalizations—and in terms specifically applicable to the engineering problem and directly useful to its solution.

Nothing in Mr. Wahlstrom's paper can be construed to oppose this viewpoint; it may be inferred from the paper's general context that the author would agree fully. However, the paper is brief and is purposely concerned, in the main, with elementary fundamentals; it would be easy to derive the erroneous conclusion (from the few concepts and examples cited, and from the simple geological principles described) that the scope of geology in tunneling is more limited than is actually the case. In reality, the applications of geology to tunneling are manifold and complex; its application to tunneling problems and operations is a matter of long standing, although it has become greatly expanded in the last one or two decades.

The paper describes, for example, the association of soft or squeezing ground with the geological processes of chemical alteration, and the "arching" effect of caving ground. Nevertheless, if an uninitiated reader is to be protected from gaining the mistaken impression that the geologist can supply no more than such academic explanations of tunneling problems, allusion must be made to other important considerations. For instance, it should be mentioned at least that the mechanics of such matters have been comprehensively analyzed by engineering geologists and engineers. Through these analyses such geological phenomena have been translated into specific stress-strain interpretations, and the interpretations, in turn, have been applied quantitatively to answer such questions as: How much and what kind of support will be required? Will permanent lining be required—what kind and how strong?

Such questions may seem to some to be beyond the proper sphere of a geologist's interest, but it is essential in any description of the relationship of geology to engineering to point out that the engineering and geological aspects of these problems are merged and interlocking and cannot be disentangled. The engineer must, of course, retain responsibility for the decision as to what kind of structure is most appropriate, and how it is to be built.

The geologist is responsible for interpreting the geologic conditions to which the structure must be adapted, and for defining the problems which these conditions will present. The geologist's data and interpretations cannot be pertinent and directly useful unless he has comprehended fully the engineering viewpoint and objective, the design alternatives, and the construction procedures that are available for use—he must understand what can and what cannot be done. The geologist must be conversant at least with the technical facilities and limitations of engineering procedure if the information he obtains is to be of any practical value to the engineer. Similarly, the geologic studies must be intimately coordinated with the economic considerations, which,

in the long run, dictate what can or cannot be done within the controlling limits of cost and value.

Simply stated, the geologic studies must be practical as well as scientifically sound. Thus, although the engineering geologist must recognize that the ease or difficulty of tunnel driving may be controlled to a large extent by the relationship between the direction of the tunnel and the direction and inclination of the schistosity, foliation, or stratification of the rocks (as succinctly described in the paper), he must also recognize that, in practice, it is only occasionally practicable to vary the course of a tunnel very much from a predetermined line to take advantage of such factors. Similarly, although the theoretical advantage of skewed or asymmetric tunnel sections shaped to conform with the structure of the rock is well described in the paper, the geologist should be equally aware of possible, serious engineering disadvantages to such shapes.

These disadvantages, even though commonly of negligible importance in mining tunnels, are frequently critical in the tunnels of the civil engineer, where the shape is often predetermined and controlled by factors directly related to the function of the tunnel. For example, in unlined tunnels designed to carry water, overbreak or irregular shape may be seriously detrimental to hydraulic efficiency, and may even so reduce the efficiency that a larger tunnel diameter is required with attendant increased costs. If the tunnel is to be lined, and has been designed with a shape and diameter controlled by the capacity requirements, any overbreak represents so much rock needlessly removed and replaced by expensive concrete and steel. The geologist cannot remain ignorant of these considerations but must take them into account specifically if the geologic studies are to be of maximum practical use to the engineer.

Although the paper acknowledges the desirability of applying geology to actual tunneling operations and recommends periodic geologic examinations and the assembly of geologic records as the work progresses, the reader may get the impression that the author is primarily concerned with the exploration of prospective sites. The experience of the United States Bureau of Reclamation attests to the value of thorough explorations continuously coordinated with the progress of design studies; on the basis of that experience—not only with tunnels, but also on engineering undertakings in general—one could recommend even stronger emphasis on drilling, ground-water testing, geophysical explorations, and the application of any other technique for searching beneath the earth's surface and securing useful reliable data bearing on the engineering plan.

It is difficult to recall a structure site that has been "overexplored"; money spent on judicious exploration is inevitably saved many times over in cheaper and more efficient design and construction. However, the applications of geology during construction are at least as important and, in some respects, more productive of useful results. The preceding comments have stressed the necessity for quantitative geological determinations, but it is recognized that this quantitative ideal is usually unattainable during the stage of exploration—no matter how many holes are drilled, or how exhaustively routine and special

investigational techniques are applied. At best, explorations serve only to reduce the guesswork and increase the assurance of advance predictions.

During construction, when the rock is exposed for precise study and when the water flow can be measured and interpreted, the measurements, observations, and tests necessary for quantitative appraisals can be made. Moreover, when precise studies made during construction (particularly if compared closely with the predictions made during the explorations) become the subject of comprehensive records, they serve as the principal method by which the procedures of tomorrow may profit by the experience of yesterday and today. The literatures of both engineering and geology need to be augmented by more "case histories," carefully and fully described.

The paper suffers from two important omissions: No mention is made of the problems of (1) residual rock stress, so important in relation to rock bursts and "heaving ground," which is sometimes critical to the design of tunnel linings, and (2) rock temperature, so important because of its technical and economic implications with respect to ventilation. The extent of recent researches in both problems attests to the current interest of engineers and geologists in these important considerations. Related minor or special problems, such as the occurrence of gas or hot water, should be mentioned even though space limitations have prevented their thorough discussion by both Mr. Wahlstrom and the writers.

There is a dearth of critical, fundamental discussions of the geological principles applicable to tunneling problems. Such data and written discussion as may apply are widely scattered and not easily accessible, but papers of the type presented by Mr. Wahlstrom represent an encouraging step toward the rectification of that deficiency, and deserve encouragement.

Engineers and geologists desiring a more extended and analytical treatment of some aspects of tunnel geology and related construction problems may refer profitably to a treatment prepared by Karl Terzaghi,[20] Hon. M. ASCE.

[20] "Rock Tunneling with Steel Supports," by R. V. Proctor and T. L. White, The Commercial Shearing and Stamping Co., Youngstown, Ohio, 1946, p. 17.

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DISCUSSIONS

MISSISSIPPI RIVER BRIDGE AT DUBUQUE, IOWA

Discussion

BY CHARLES W. DOHN

CHARLES W. DOHN,[13] JUN. ASCE.—High secondary stresses are developed over intermediate piers in a continuous truss system such as that used on the Dubuque bridge. This is an unusual condition following the strict interpretation of accepted specifications.[14,15,16] An-analysis for secondary stresses, particularly over the intermediate piers, is thus mandatory for this rather unusual long span continuous design.

A complete discussion of the design of this bridge should include a detail of a typical joint connection, particularly near the intermediate piers. Relatively rigid joint connections, stiff members, and large deflections contribute to high secondary stresses; when these are added to all other effects, including wind, pier settlement, and temperature, they could easily become critical.

It would seem that using truss members of different materials would also contribute to large secondary stresses at some points. A silicon steel tension member designed for the same total stress as a carbon steel tension member will deflect 1.33 times as much as the carbon steel member. Such deflection could magnify secondary stress conditions. Why was silicon steel specified for some members and carbon steel for others?

The specifications previously mentioned[14,15,16] state:

> "Secondary stresses due to truss distortion or floor beam deflection usually need not be considered in any member the width of which measured parallel to the plane of distortion is less than one-tenth of its length."

Originally the speicfications were those of the American Railway Engineering Association, dated 1920. It appears that most of the members meet this condition. However, the use of members of different materials, the long and unu-

NOTE.—This paper by R. N. Bergendoff and Josef Sorkin was published in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by Jonathan Jones; and October, 1947, by A. Floris, and T. H. Rust.

[13] Engr., New York, N. Y.

[14] "Standard Specifications for Highway Bridges," A.A.S.H.O., Washington, D. C., 1944, p. 161, article 3.6.7.

[15] "Specifications for Steel Highway Bridges," A.R.E.A., Chicago, Ill., 1927, article 1409.

[16] "Report of Committee XV on Iron and Steel Structures," A.R.E.A., Chicago, Ill., January, 1920, clause 47.

sual continuous span, and the large deflections would seem to indicate the need for further study.

It might be possible to increase the strength of this bridge greatly by altering certain members subject to high secondary stresses. The importance of secondary stresses in modern truss structures has been somewhat neglected in designing ordinary, standard, short span trusses after initial investigations proved their relative values. The tendency has been to assign blanket values, or to rely on a wide margin of safety factor in design. Unusually long span trusses, trusses of unusual design, and continuous trusses still warrant analysis for secondary stresses. Extreme reversals of secondary bending under moving loads might cause yielding leading to fracture at the joints of a member or, after a limited number of reversals under high stress, the member might exhibit fatigue failure.

A method extensively used to provide for secondary stresses in continuous bridges, such as the Dubuque bridge, is to rely on methods of geometric cambering to neutralize the secondary stresses. However, the loading for maximum direct stress is not necessarily the same for maximum secondary stress or maximum total stress. Influence lines for secondary bending stress in each member due to unit loads can be used to determine absolute maximum total stress. Secondary bending stress should be determined for at least the members in the vicinity of the intermediate supports of continuous truss bridges. The use of a trussed arch span with inherent increased deflection would seem conducive to increased secondary stresses.

Rough calculations indicate possible maximum secondary stresses approximating maximum primary stresses. With geometric camber such stresses cannot be entirely neutralized. Increases are allowable for combinations of secondary and axial stresses and $33\frac{1}{3}\%$ is a common value. Since 25% overstress has already been allowed for dead load plus live load plus a 30-lb wind, only $8\frac{1}{3}\%$ remains for secondary stress which is not enough to absorb indicated values for some members even after deducting the neutralizing effect of possible geometric camber. Whether failure could appear as fatigue failure is problematical. Reversal of secondary bending stress plus reversal of primary stress in a member would be particularly contributory to fatigue failure and a reduction in working stress rather than an increase would be in order. It appears that liberties have been taken with the factor of safety with questionable economy considering the further possibility of pier settlement and aerodynamic effects. Light earthquake shocks have been reported in the general region.

The Dubuque bridge presents a neat outline and the members are nicely proportioned and arranged. The tabulation of stresses is concise but lacks a column for secondary stress or a statement as to geometric camber. In closing the writer wishes to endorse heartily the opening paragraphs of the discussion by Jonathan Jones, M. ASCE.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CONSOLIDATION OF FINE-GRAINED SOILS BY DRAIN WELLS

Discussion

BY WALTER KJELLMAN

WALTER KJELLMAN,[26] ESQ.—The consolidation of a fine-grained soil subjected to a load can be accelerated by vertical drains inserted in the soil. In the United States such drains consist of circular sand-filled wells, having normally about a 20-in. diameter and a spacing of from 10 ft to 15 ft. In Sweden, band shaped cardboard wicks of $\frac{1}{8}$-in. by 4-in. cross section (Fig. 11) are used. These are furnished with inner longitudinal channels and spaced about 4 ft apart. It may perhaps interest readers to learn how the problems, so clearly dealt with in this paper, are looked upon in Sweden.

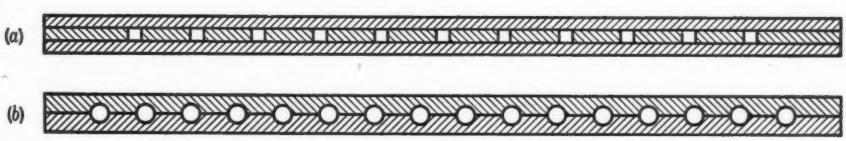

FIG. 11.—CROSS SECTION OF BAND SHAPED CARDBOARD DRAIN

(a) Present Type

(b) Prospective Type

Swedish investigations concerning "deep drainage" of soils were started in 1936. Owing to arching of ground and overburden in all practical cases, the momentary strains were deemed to be nearly equal everywhere in the ground. Therefore, all Swedish calculations refer to this very simple case of equal strain. In the United States, on the contrary, all calculations of this kind prior to Mr. Barron's paper seem to refer to the very complicated case of free strain. Fig. 8 shows that both cases give nearly the same result. Consequently, there is no reason for anyone to become further involved with free strain.

NOTE.—This paper by Reginald A. Barron was published in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1948, by Kenneth S. Lane; and February, 1948, by Philip Keene.

[26] Head, Statens Geotekniska Institut (The Royal Swedish Geothechnical Institute), Narvavägen 25, Stockholm, Sweden.

When determining the spacing of the drains in the United States, great reliance is placed on the horizontal draining effectuated by coarse-grained layers. In Sweden such layers exist in the lower part of the glacial clay deposits, but they are of little use since the spacing must be determined with regard to the upper part of the deposits (the upper part of the glacial clay and, on top of it, the postglacial clay), which as a rule contains no coarse-grained layers. Furthermore, the fine-grained soils in Sweden seem on the whole to be considerably less pervious than those in America. For these reasons, and because the equivalent radius of the wicks used in Sweden is much smaller than the radius of the American wells, the spacing is considerably smaller in Sweden.

By laboratory tests it has been found that the permeability of a clay without coarse-grained layers is not appreciably reduced by remolding. The unfavorable effect of peripheral smear, discussed by the author, must therefore refer to coarse-grained layers only. No reliance is placed on such layers in Sweden, so that peripheral smear need not be considered.

In Sweden the spacing of the drains is rather small. Thus, the vertical water flow is unimportant when compared to the radial flow and is disregarded. This approximation, which simplifies the calculation and leads to safe results, is justified by the uncertainty of the calculation.

In principle the triangular drain pattern is the most economical one, as stated by Mr. Barron. However, certain considerations have shown that in this respect the difference between the triangular and square patterns is quite unimportant. Therefore, in order to attain certain small advantages when driving the wicks, no fixed pattern is used, other than to make the spacing of the wicks in each row equal to the distance between rows.

The longitudinal channels in the cardboard wick easily can be made numerous enough and wide enough to let through (without appreciable resistance) any water flow that the one drain may be called on to carry. Furthermore, the permeability of the cardboard is very great when compared to that of the fine-grained soils in question. Therefore, there is no need to complicate the calculations by taking into account the flow resistance in the drain. Because the cardboard serves as a perfect filter, there is, of course, no risk of the channels becoming clogged as may occur in sand-filled wells. Laboratory tests have shown that the channels do not collapse even under a very high clay pressure on the wick. This is probably due to arching in the clay, causing the pressure to be much lower on the covers of the channels than between them.

The question has been raised as to whether a well (like a pile) attracts a great part of the load, and whether it can be damaged when the soil settles. A cardboard wick cannot, of course, affect the load distribution. Tests have shown that it may crease, if the vertical compression is great, but that it is not damaged.

In Swedish practice (as follows from the foregoing statements), the time-settlement curve of a deep-drained soil is calculated in the simplest possible manner—assuming equal strain, no coarse-grained layers, radial flow only, no peripheral smear, and no drain resistance. The calculation, made in 1937 but never published, is contained in principle in Mr. Barron's paper. As a result of

the calculation the following procedure is used for determining the spacing of the drains (square pattern):

(*a*) Having assumed a time, t_h, in months, in which a certain percentage of the final settlement is to occur, Fig. 12 is used to find the value of g, which is a quantity of calculation with no physical meaning.

(*b*) Knowing the value of g, and also the coefficient of consolidation of the soil, c_h (in square centimeters per second), the value of m, which is a quantity of calculation with no physical meaning, is computed from

$$m = g\, c_h \quad \text{.........................} (92)$$

(*c*) Knowing the value of m, Fig. 13 is used to find a convenient combination of drain radius r_w and spacing S.

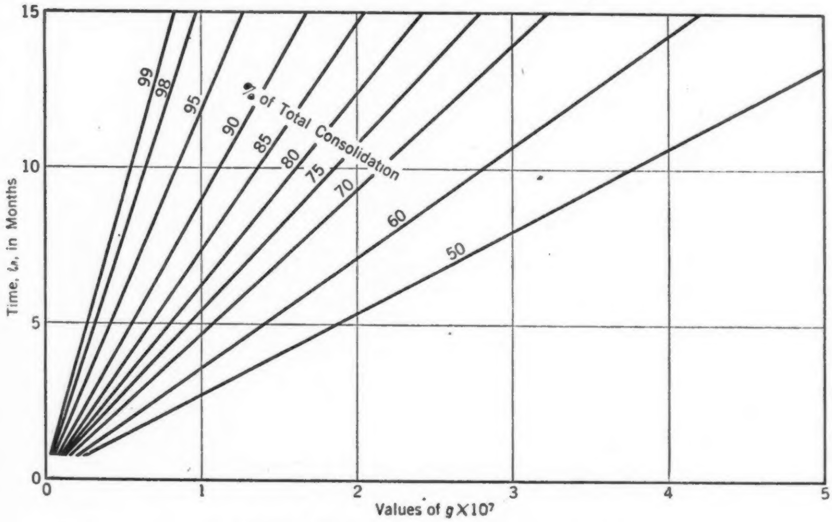

FIG. 12.—DIAGRAM FOR FINDING VALUES OF g

This procedure may be used in cases not too divergent from Swedish conditions.

Obviously, the draining effect of a drain depends to a great extent upon the circumference of its cross section, but very little upon its cross-sectional area. Therefore, the circular cross section has been abandoned in Sweden in favor of the band shaped drain. Certain considerations show that the cardboard wick is as effective as a circular drain with a 1-in. radius. (The circumference of the latter is slightly greater than that of the former.) Thus, the curve in Fig. 13 for a 1-in. radius can be used for the wicks. The curve for a 10-in. radius is valid for the sand wells with a 10-in. radius frequently used in the United States. Comparing these two curves, it appears that wells with 10-ft spacing are equivalent to wicks spaced 6.2 ft apart. Thus, in this case, one well is equivalent to 2.5 wicks.

The wick has several advantages over the well. First, very little material is consumed in its manufacture, and, second, as both its weight and its volume

per unit of length are small and as it can be rolled on a drum, it is easy to handle. This means that production can be concentrated in a factory, whence the wicks can be transported inexpensively to the different sites and driven into the ground by a machine working at high speed in about the same manner as a sewing machine.

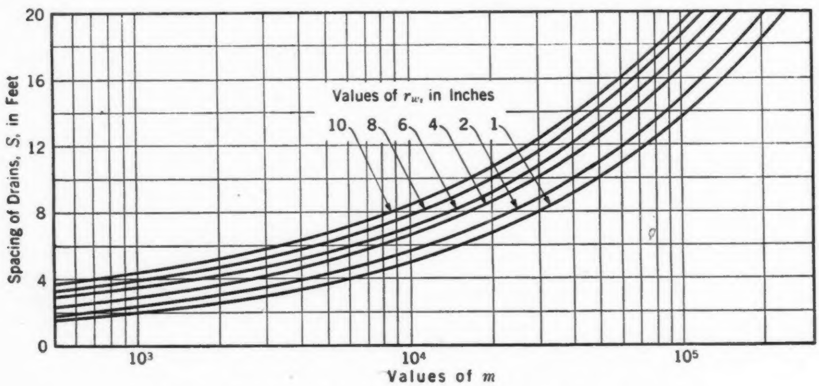

FIG. 13.—DIAGRAM FOR FINDING THE SPACING OF DRAINS

At the Halmsjön Airport outside Stockholm (Sweden) some 3,000,000 ft of cardboard wicks are to be driven in 1947–1949. The cost is between 10¢ per ft and 15¢ per ft. Further information about the wick method will appear in the *Proceedings* of the Second International Conference on Soil Mechanics to be held in Rotterdam, Holland, in 1948.

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DISCUSSIONS

DEVELOPMENT AND HYDRAULIC DESIGN, SAINT ANTHONY FALLS STILLING BASIN

Discussion

BY FRED W. BLAISDELL

FRED W. BLAISDELL,[10] Assoc. M. ASCE.—The thoughtful and constructively critical comments of the discussers are evidence that the writer's aim of presenting the results of the SAF (Saint Anthony Falls) stilling basin investigation in such a way that each reader could independently evaluate and intelligently discuss the design rules developed therefrom was achieved. The discussions could not have been prepared without the expenditure of considerable time and effort, and the writer wishes to acknowledge this work and to express his sincere appreciation of the contributions of the discussers. They have materially increased the value of the paper to the engineering profession.

Of considerable interest are the energy losses and relationships in the hydraulic jump and stilling basin computed by Mr. Blotcky and summarized in Table 10. The proportion of the energy at Section 1, Fig. 18, that is lost in the hydraulic jump is least at small values of $\mathbf{F}_1$ (15*e*).[10a] This may be why the blocks are most effective at low values of $\mathbf{F}_1$, as Mr. Blotcky has noted. However, Col. 9, Table 10, and the results of the original tests as recalled by the writer seem to indicate that the energy loss ascribed to the blocks is far less than is warranted by their importance. For example, in Table 10 the median value of ϵ_b/ϵ_j is 0.11. For this value of ϵ_b/ϵ_j the energy loss due to the blocks alone is only 6% of the energy at Section 1, Fig. 18. Since the reduction in basin length achieved through the use of blocks and sills is in the order of 75%, it may be that the blocks have some function other than the dissipation of energy. However, it is probable that the location of Section 2′, as assumed by Mr. Blotcky, is incorrect. At the end of the SAF basin the depth of flow

NOTE.—This paper by Fred W. Blaisdell was published in February, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by Donald E. Blotcky, M. M. Culp, Paul Baumann, A. J. Peterka, and Louis M. Laushey; and December, 1947, by J. H. Douma, and Robert F. Ewald.

[10] Project Supervisor, U. S. Dept. of Agriculture, SCS, St. Anthony Falls Hydr. Laboratory, Minneapolis, Minn.

[10a] Numerals in parentheses, thus: (15*e*), refer to corresponding items in the Bibliography (see Appendix I of the paper), and at the end of discussion in this issue.

is increased by the boil and decreased by the end sill. Furthermore, the streamlines have considerable curvature at this point, and, as a result of the surface roller, there may be upstream components of the velocity at this section (see Fig. 1(*b*)). An additional reason why the location of Section 2′, Fig. 18(*b*), is incorrect is the fact that some energy dissipation occurs downstream from the end of the basin. Section 2′ should be selected at some point in the downstream channel. The writer has no data to indicate the depths and velocities at a point where normal flow conditions have been established in the downstream channel. In the absence of this information, Mr. Blotcky's computations summarized in Table 10 are accepted for their relative value.

These comments regarding Section 2′ also apply to the coefficient of form drag (C_d) computations summarized in Fig. 23. Although the magnitudes of C_d shown in Fig. 23 are probably inaccurate, Mr. Blotcky is to be commended for his most interesting analysis in which he introduces the effect of the floor blocks into the momentum equation for the hydraulic jump.

The use of blocks certainly modifies Eq. 2, but there is some question as to whether or not their use invalidates it completely, as Mr. Blotcky maintains. In fact, in commenting on Fig. 21 he notes that certain ratios computed for the block jump "are comparable to those in a hydraulic jump * * *."

In view of Eq. 9 and of Mr. Blotcky's pertinent comments, it would seem that the use of d_c, in place of d_2 as used by the writer, is a matter of personal preference. The use of d_c was considered, but this parameter was not adopted because of the facts expressed in Eq. 9 and because its use would require the computation of an additional parameter when designing the SAF basin, without any apparent advantage being gained thereby.

In Fig. 19 Mr. Blotcky has used d_c as a parameter when replotting some of the data presented in Table 2 and Fig. 7. Through the data points he has drawn the curve represented by Eq. 11. The relationship between d_c and d_2 embodied in the hydraulic jump equation makes it difficult to compare mathematically Eq. 11 with Eq. 3. It is, however, possible to express Eq. 11 in terms of d_1. Substituting for q^2 in Eq. 10 its equivalent, $\mathbf{F}_1\, g\, d_1\, d^2_1$,

$$d_c = \sqrt[3]{\mathbf{F}_1\, g\, d_1\, d^2_1/g} = \sqrt[3]{d^3_1\, \mathbf{F}_1} = d_1\,(\mathbf{F}_1)^{0.33} \qquad (41)$$

Substituting Eq. 41 in Eq. 11,

$$\frac{L_B}{d_1\,(\mathbf{F}_1)^{0.33}} = \frac{6.2}{(\mathbf{F}_1)^{0.25}} \qquad (42)$$

or

$$\frac{L_B}{d_1} = 6.2\,(\mathbf{F}_1)^{0.08} \qquad (43)$$

B. A. Bakhmeteff, Hon. M. ASCE, and A. E. Matzke, Assoc. M. ASCE (15*f*), have shown that the relationship L/d_1 between the length of the hydraulic jump, L, and d_1 varies considerably with λ, or $\mathbf{F}_1$ as it is designated by the writer, whereas the ratio L/d_2 is approximately constant. Inasmuch as the action in the SAF stilling basin is considered to be a forced hydraulic jump and the basin length is related to the jump length, the writer prefers the ratio L/d_2.

Mr. Blotcky attempts to compare Eqs. 11 and 3 in Fig. 19. The writer strongly objects to this comparison. The comparison should be between the curve and its pertinent data, which are given for Eq. 3 in Fig. 7 and for Eq. 11 in Fig. 19. The writer has computed L_B by Eqs. 3 and 11 and has tabulated the results in Table 12, using tests selected from Table 6. The values of L_B listed in Table 6 were those used in the tests. These values have been recomputed by inserting in Eq. 3 the observed values of d_1, d_2, and $\mathbf{F}_1$, which differed slightly from values used in the design of the model. It will be noted in every case except two that L_B is shorter when computed by Eq. 11 than when Eq. 3 is used. In some cases the computed basin length is 20% shorter. This difference is great enough to make the use of Eq. 11 inadvisable. Although based on the same data, Eq. 3 has been thoroughly checked by model tests and its use is to be preferred.

On the basis of Fig. 20, the writer agrees with Mr. Blotcky that the form of Eq. 18 is more desirable than that of Eqs. 6. However, most of the data

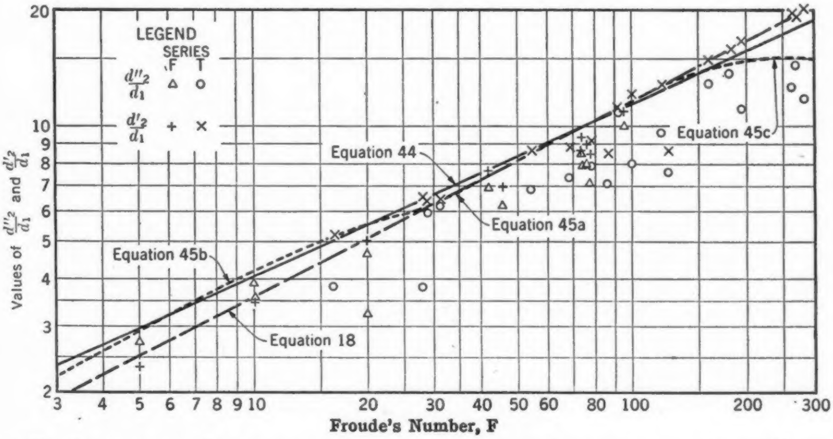

FIG. 39.—DIMENSIONLESS PLOT SHOWING TAILWATER DEPTH AS A FUNCTION OF THE FROUDE NUMBER

presented in Fig. 11 are not included in Fig. 20. The writer has recomputed the data shown in Fig. 11 and summarized them in Table 13. These data are plotted in Fig. 39, in which the curve shown has the equation:

$$\frac{d'_2}{d_1} = 1.4\ (\mathbf{F}_1)^{0.45} \qquad (44)$$

Eq. 18 is also shown to permit comparison. The writer considers that Eq. 44, which is based on better data than is Eq. 18, is to be preferred, but that either Eqs. 6 or Eq. 44 will give good results. This contribution by Mr. Blotcky simplifies the design of the SAF stilling basin and is greatly appreciated by the writer. Mr. Blotcky's significant contribution in this instance shows the value of publishing papers in *Proceedings* where they may receive the consulting services of the entire membership of the Society.

TABLE 12.—COMPARISON OF BASIN LENGTHS COMPUTED BY EQS. 11 AND 3

Test No.	q	d_c	d_1	$\mathbf{F}_1$	L_B		Cols. 5/6
					Eq. 11	Eq. 3	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
T53	4.00	0.792	2.82	288	1.19	1.48	0.80
T63	9.42	1.40	4.875	252	2.18	2.68	0.81
T61	9.94	1.45	4.826	196	2.41	2.92	0.83
T52	4.00	0.792	2.410	121	1.48	1.75	0.85
T60	14.4	1.86	5.48	100.6	3.64	4.27	0.85
T59	14.8	1.90	4.98	54.6	4.32	4.91	0.88
T58	14.9	1.90	4.39	28.2	5.12	5.55	0.92
T54	15.0	1.91	3.96	16.5	5.88	6.14	0.96
T57	20.0	2.32	4.50	12.4	7.66	7.78	0.98
T56	21.0	2.39	4.09	7.0	9.12	8.79	1.04
T55	21.0	2.39	4.05	6.7	9.22	8.85	1.04

TABLE 13.—SUMMARY OF DATA ON TAILWATER DEPTH

Test No.	$\mathbf{F}_1$	d'_2/d_1	d''_2/d_1	Test No.	$\mathbf{F}_1$	d'_2/d_1	d''_2/d_1	Test No.	$\mathbf{F}_1$	d'_2/d_1	d''_2/d_1
	(1)	(2)	(3)		(1)	(2)	(3)		(1)	(2)	(3)
F9	96	10.80	10.00	T39	183.1	15.77	13.57	T49	68.7	8.63	7.29
F17	74	9.22	8.50	T40	29.2	6.29	5.88	T50	121	12.83	9.59
F18	74	8.58	7.88	T41	31.8	6.39	6.14	T53	288	19.98	11.83
F20	76	8.85	7.98	T42	93.7	11.06	10.87	T54	16.5	5.12	3.73
F21	42	7.58	6.92	T44	161	14.72	12.95	T58	28.2	6.50	3.73
F28	78	8.43	7.07	T45	87.0	8.47	7.02	T59	54.6	8.59	6.77
F30	46	6.83	6.20	T46	274	19.16	14.40	T60	100.6	12.02	7.90
F94	20	4.97	4.66	T47	126	8.51	7.57	T61	196	16.60	11.08
F97	10	3.40	3.53 $<$	T48	78.5	9.10	7.80	T62	267	19.57	12.52
F98	5	2.29	2.70 $<$								
F99	10	3.40	3.85 $>$								
F101	20	4.96	3.22 $>$								

The writer has changed the parameters of Eqs. 6 into those of Eq. 44 by substituting for d_2 its equivalent given in Eq. 2*b*. Eqs. 6*a*, 6*b*, and 6*c* then become

$$\frac{d'_2}{d_1} = 0.425\left(-1 + \sqrt{8\,\mathbf{F}_1 + 1}\right) \qquad (45a)$$

$$\frac{d'_2}{d_1} = \left(-1 + \sqrt{8\,\mathbf{F}_1 + 1}\right)\left(0.55 - \frac{\mathbf{F}_1}{240}\right) \qquad (45b)$$

and

$$\frac{d'_2}{d_1} = \left(-1 + \sqrt{8\,\mathbf{F}_1 + 1}\right)\left(0.50 - \frac{\mathbf{F}_1}{1{,}600}\right) \qquad (45c)$$

respectively. Eqs. 45 have been plotted in Fig. 39 to permit their comparison with Eq. 44.

The writer cannot agree with Mr. Blotcky that the undular type of jump occurs in the block jump simply because d'_2/d_2 is greater than one for small values of $\mathbf{F}_1$. It is the recollection of the writer that the roller associated with the direct jump occurred at all values of $\mathbf{F}_1$ at which tests were made. Because a slope exists at the entrance to the SAF stilling basin, it is believed that in-

creasing d'_2/d_2 is conducive to causing the entering stream to plunge under the tailwater, as it does for the direct jump, rather than to ride at the tailwater surface as it does for the undular jump. In any event, it is difficult to follow Mr. Blotcky's reasoning as to why Eq. 18 is invalid when $\mathbf{F}_1$ is less than 15. The writer feels that the form of Eq. 18 is excellent and represents the complete data at all values of $\mathbf{F}_1$, but that the constants used in Eq. 44 are to be preferred.

Mr. Culp followed closely the model studies leading to the development of the SAF stilling basin design and contributed freely from his experience in the design and construction of soil conservation structures. In matters regarding the feasibility of certain hydraulic features from the viewpoint of the structural designer and construction engineer, his advice was frequently sought, freely given, and invariably followed. It is a pleasure to be able to acknowledge his contribution publicly. Under the direction of Edwin Freyburger, chief of the Soil Conservation Service regional engineering division in Milwaukee, Wis.,

FIG. 40.—STUART PERRY SPILLWAY, CRAWFORD COUNTY, IOWA, PRIOR TO STORM OF JUNE 22, 1947

Mr. Culp designed the first SAF stilling basin before the final report was prepared. Since then he has designed and built several others. It was hoped that he would be able to report on some structures that had been tested by nature at about the design flow. However, since his discussion was written, the Stuart Perry spillway located in Crawford County, Iowa, has received a severe test. Fig. 40 shows a view of this structure taken prior to the storm described in a letter from Mr. Culp, who obtained his information from Floyd Nimmo, construction engineer on the job.

"The storm occurred June 22, 1947, between 4:00 p.m. and 6:00 p.m. with most of the rainfall in the first hour. No standard or automatic gauges were in the vicinity, but rough measurements in buckets, etc. indicated a total rainfall of about seven inches. This storm produced a maximum head on the inlet of the spillway of almost four feet, or nearly a foot more than the design head. Hence, even though the inlet was somewhat restricted by poor placement of the earth fill * * * the flow through the spillway was probably equal to or greater than the design discharge [more

than 50% in excess of the design discharge, assuming a 3 ft design head, a 4 ft actual head and unrestricted approach to the inlet]. * * * Mr. Nimmo tells me that the outlet functioned perfectly and that there was very little erosion in the immediate vicinity of the outlet."

Fig. 41 shows views of this structure taken on February 18, 1948. It is apparent that very little scour occurred as a result of the flood of June 22, 1947. The writer has seen three additional outlets which he has every reason to believe have passed heavy flows. These apparently functioned as anticipated, although the downstream channels had not eroded to a stable grade at two of the outlets when the writer saw them in 1945.

Mr. Baumann believes that it would have been helpful to include information regarding the energy gradient between the reservoir and the stilling basin. The SAF stilling basin will be used with drop inlet culverts as well as with overflow spillways. A different value of the coefficient ϕ would have to be determined, not only for each type of structure, but also for each size of structure. To provide a limitation on the scope of the study, it was assumed that flow conditions were known at the stilling basin entrance and at a point where normal conditions are reestablished in the downstream channel; the tests reported in this paper cover only the assumed unknown region between these points. In many cases ϕ, a measure of the lumped-together losses up to the basin entrance, can be computed with sufficient precision from information presently available to the designer. High precision is unnecessary in designing a SAF stilling basin. This comment is no argument for sloppy work, but is rather a recognition of the fact that the magnitude of many of the factors involved can be determined only roughly. The foremost of these factors is probably the rate of runoff on which all subsequent design depends. It is certainly very difficult to determine the rate of runoff with any high degree of precision.

Mr. Baumann's attempt to determine d'_2 from a consideration of the amount of kinetic energy transformed into heat energy is most interesting, but it appears that additional work is required before this method can be used without resort to model studies. However, any attempt to develop an equation from theoretical considerations is always commendable and leads to a better understanding of the problem.

The writer shares most of the views expressed by Mr. Peterka. His comments are a welcome addition to the paper. Nevertheless, the writer cannot agree with Mr. Peterka's comment that the chute blocks and floor blocks of the SAF stilling basin can be dispensed with. It is true that much of the energy dissipation takes place in the channel downstream from the stilling basin. It is also true that the SAF stilling basin throws water upward and away from the structure as does the Fontana Dam (Tennessee Valley) energy dissipator, but the water does not reach a height sufficient to create spray except at the surface under some conditions. The action is such that energy dissipation beyond the basin exit takes place principally near the water surface, as at Fontana Dam, rather than at the bed. However, some energy is dissipated within the SAF basin, in contrast to the Fontana Dam distributor where all energy dissipation takes place beyond the deflector bucket. At high values of **F** a bucket could be used in place of the SAF stilling basin, but at low values of **F** there is in-

sufficient energy available to throw the jet into the air. The writer feels that there is a need for both types of energy dissipator but that the "apron accessories" are required and are economical in the SAF design.

Mr. Peterka voices a common objection to blocks and piers, and the writer feels somewhat the same. It is significant, however, that piers are used almost invariably when the problem of energy dissipation is particularly difficult, or when the cost must be kept at a minimum. Such considerations were uppermost in the minds of those engaged in developing the SAF design.

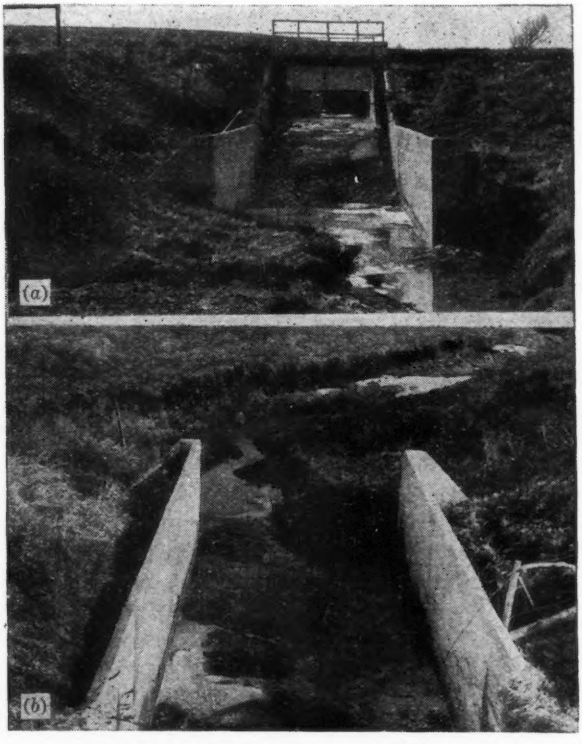

FIG. 41.—STUART PERRY SPILLWAY ON FEBRUARY 18, 1948
(a) Upstream View (Compare with Fig. 40)
(b) Downstream View; Bank Scour Occurred Before Spillway was Built

Whether or not special protection from wave action is required in the downstream channel depends to some extent on the frequency with which the design flow is expected and on the duration of such flow. Little wave action was apparent in the models until the discharge approached the design value. At lesser discharges the surface disturbance might be described as "ripples." For soil conservation structures the commonly used design storm has an anticipated 50-yr frequency. For small watersheds the design flow occurs for only a few hours at a time. Under these conditions vegetation is undoubtedly sufficient to protect the banks. The writer shares Mr. Peterka's opinion that for structures subjected to the design flow for long periods, some bank protec-

tion from waves will probably be required. Whether it is more expensive to provide bank protection from wave wash for maximum flows or to use a larger basin, as suggested by Mr. Douma, is a problem that the designer of each structure must solve. In many cases the value of the land protected by riprap or other methods may be less than the cost of the protection. It should be noted that, after a stable condition has been achieved in an erodible channel, the normal channel is wider at the basin exit than it is farther downstream. This phenomenon should be taken into consideration if bank protection is used. Otherwise the bank protection would be subject to excessive velocities and might possibly be damaged.

The writer feels that Mr. Peterka need not be concerned about the high surface velocities downstream from the apron. Since the highest velocities are on the center line of the channel and are near the surface, it is not believed that protection is required for this reason alone.

Mr. Peterka suggests that steeper slopes at the basin entrance probably contribute to increased basin efficiency. The slopes used during the SAF tests varied from 1 on 2.6 to 1 on 1.5. In the SAF basin, however, the chute and floor blocks aid in producing a uniform velocity distribution at the basin exit and may offset the poorer results obtained by Mr. Peterka for the Horsetooth Dam outlet works in Colorado when the entering slope was decreased.

The range of slopes used at the entrances to the SAF stilling basin models is too narrow to permit a determination of the effect of changes in the transition profile on the proportions of the chute blocks. The writer can therefore offer no evidence to substantiate or to refute the explanation offered by Mr. Peterka, which he based on "pure conjecture" and on the results of tests on the Enders Dam spillway in Nebraska.

During the SAF stilling basin tests, it was observed that the use of chute blocks permitted a lowering of the tailwater without washing the jump out of the basin. Mr. Peterka also found this to be the case at Enders Dam. The writer found, as did Mr. Peterka, that a higher end sill helped keep the jump in the basin. However, the SAF end sill is subjected to higher velocities than is the Enders Dam sill; its height has a greater effect on the flow; and the end sill could not be raised without greatly increasing the boil height, which would necessitate raising the side walls. All these considerations were kept in mind when it was finally decided that the best solution for the SAF stilling basin was to leave the chute blocks in place and to determine the best end sill height for the assumed form of the stilling basin.

In Fig. 34 Mr. Laushey has attempted to compare d'_2/d_2 for the SAF stilling basin with certain results he obtained. The writer feels that there is only one point of comparison—at the intersection of Eq. 6*b* and Mr. Laushey's curve for $d'_2/d_2 = 0.07$. This intersection indicates only the Froude number (or d_1/d_c) at which the two basins have the same tailwater depth. The limiting curves labeled "SAF; d'_2/d_2 (no factor of safety)" and "limiting curve, rectangular sill" are not based on the same criteria and are therefore not comparable.

The writer does not believe that the height of the end sill in the two types of stilling basin can be compared as Mr. Laushey has done. Fairly high ve-

locities existed at the end sill in the SAF basin, whereas in Mr. Laushey's basin the velocities were, without doubt, comparatively much lower. In the second place, the SAF basin is much shorter. These considerations and others, in addition to the effect of the blocks mentioned by Mr. Laushey, undoubtedly contribute to the differences in the recommended values of d'_2/d_2.

Mr. Laushey's curve for zero end sill height (Fig. 36) gives a basin length at variance with the length of jump determined by Messrs. Bakhmeteff and Matzke. Fig. 36 does, however, give an excellent picture of the basin length reduction that can be achieved through the use of sills, and also clearly shows the further reduction made possible by the block arrangement of the SAF stilling basin.

Mr. Laushey notes that in the SAF stilling basin the tailwater depth had to be increased at values of $\mathbf{F}_1$ less than 30, and states that a similar increase was required for the end sill basin. However, it can be seen in Fig. 34 that for a constant end sill height the relative tailwater depth is increased with increasing values of the Froude number for the end sill basin. Just the opposite is the case for the SAF basin, where Fig. 11 shows that the relative tailwater depth is increased with decreasing values of $\mathbf{F}_1$ when $\mathbf{F}_1$ is less than 30.

The results presented by Mr. Laushey are most interesting and valuable. Rather than deprecate his results, the writer has attempted to show that the proper magnitudes of the parameters are so dependent on the type of basin under consideration that it is not possible to make a valid comparison between many of the common elements making up these two types of basin. In this the writer agrees with Mr. Douma's statement, "* * * hydraulic design engineers should proceed with caution in applying these rules to other types of stilling basins." Mr. Peterka also has expressed somewhat the same thought in his discussion.

Mr. Douma objects to the writer's definition of $\mathbf{F}_1$ and prefers λ. There are three reasons why the writer defined $\mathbf{F}_1$ by Eq. 1a: First, he learned it that way; second, it has direct physical significance, because it represents the ratio of the force required to overcome inertia to the force of gravity; and, third, this definition is used (41a) in ASCE *Manual of Engineering Practice No. 25*. Many persons use the form preferred by Mr. Douma. However, when writing in an ASCE publication, the writer feels a certain obligation to use the form approved by the Society.

Mr. Douma concludes from Fig. 37 that Eq. 39 is a satisfactory criterion for the stilling basin length when blocks and an end sill are used. The writer agrees with this statement as it applies to the United States Bureau of Reclamation (USBR) tests. He cannot, however, agree that curve (c) in Fig. 37 in itself indicates that the SAF stilling basin is too short. In the first place, Mr. Douma presents neither data nor arguments to support his statement. Furthermore, the USBR data cannot alone be used to discredit another stilling basin design based on probably as thorough and exhausting study as was the USBR design. The writer went to great length in his paper to show that the SAF stilling basin gave entirely satisfactory results "for the usual case of an erodible downstream channel," and it can be stated that the SAF stilling basin, like the USBR basin, has given satisfactory service under field conditions.

The writer feels that generalized results of tests made at public expense should be made available to the public. This is one reason why he offered the SAF results for publication; and, for this reason also, it is well that Mr. Douma has presented the design criteria for the USBR stilling basin.

It should be noted that the boil in the SAF basin rises above normal tailwater level (in contrast to larger basins where the velocity at the floor blocks is lower). Therefore, the freeboard normally used by designers should be figured from the top of the boil rather than from the tailwater surface. With this fact in mind, the freeboard proposed by Mr. Douma is in reasonable agreement with that proposed by the writer. Spray will probably go over the top of the SAF basin walls at the design flow, but masses of water, which the writer associates with splash, should not.

Mr. Douma feels that the velocity in open channels with entrained air is less than that of a corresponding nonaerated flow computed by the accepted formulas. He bases his statement on the results obtained by R. Ehrenberger (38) and on field tests made by the Bureau of Reclamation. Offsetting these results are those reported by the late L. Standish Hall, M. ASCE (11), who finds that the velocity increases when air is entrained. The most recent, and undoubtedly the most accurate, series of tests to come to the attention of the writer are those reported by Warren W. DeLapp (42). From a large number of carefully performed experiments in a 1-ft-wide channel having three different roughnesses, Mr. DeLapp shows that, for all practical purposes, normal values of Manning's n can be used when computing flows with entrained air. He concludes that the velocities with and without air entrainment will be approximately the same. However, the depth of the flow will increase with air entrainment as a result of the greater volume rate of flow. In view of Mr. DeLapp's results and the conflicting results obtained by others, the writer's assumption of identical velocities without and with air entrainment does not seem unreasonable.

Mr. Ewald has called attention to one feature of the SAF stilling basin on which the experiments are incomplete: No observations were made of the velocities or pressures at the blocks. It is true that the velocities at the floor blocks are high and that cavitation may be expected if they are high enough. Until additional data become available, the designer should proceed with caution whenever he suspects that the velocities at the floor blocks will be such that cavitation can be expected.

After the turbine room tests had been completed, the basin for test T63 was left in place and operated intermittently for purposes of demonstration over a period of 2 years. When the basin was removed, scour was observed on the wood floor in the vicinity of the floor blocks. A view of this scour is shown in Fig. 42. The erosion around the blocks was the only scour noted, its maximum depth being 0.03 in.

Mr. Ewald feels that little energy is absorbed by the floor blocks and bases his opinion on tests performed in 1914 and 1915. He tried blocks only at the beginning and at the end of the hydraulic jump, and in his words (34*b*), "* * * the middle * * * was completely neglected." The best location for the floor blocks is probably somewhere between the ends of the jump. If Mr. Ewald

had located the blocks in this region, it is probable that his present opinion would be somewhat revised. Certainly, the reduction in size of the SAF stilling basin could not have been achieved without the use of baffles and sills.

During the analysis of the SAF stilling basin data, it was noted that a significant reduction in basin size could be obtained for a given discharge if **F** could be increased. Because of this discovery, a method of increasing **F** was sought; and, as a result, a study of expanding transitions in high-velocity flows was initiated. This study, which is incomplete in January, 1948, was based on the initial assumption that the velocity would be unchanged along the transition, but that the depth of flow would be decreased. This decrease in depth causes an increase in **F**. The results to date (43) indicate that over-all savings

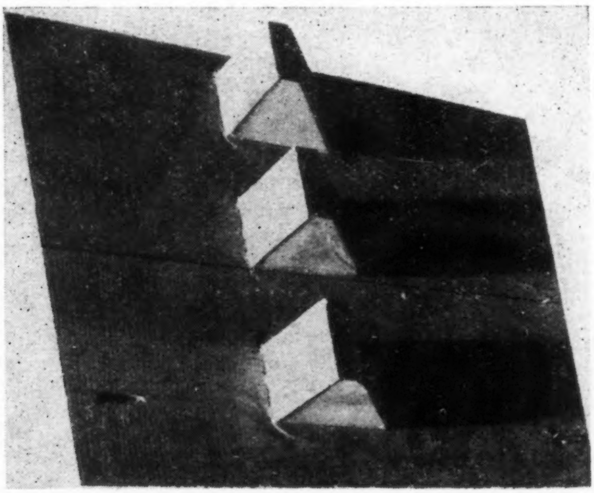

FIG. 42.—SCOUR AROUND FLOOR BLOCKS

up to 50% in concrete volume and 20% in excavation may be obtained through the use of a transition. This saving cannot ordinarily be accomplished at overflow dams, but it is possible when chutes or culverts are used. The use of a SAF stilling basin in conjunction with a transition may, under optimum conditions, result in a structure only one eighth as large as a comparable hydraulic jump stilling basin. This claim may seem to be fantastic and it may seem impossible to achieve a safe outlet structure of this size. The writer will not attempt to prove his contention here, but suggests that any person interested make the computations himself, and, if he doubts the adequacy of his design, subject it to verification by model tests.

The writer may have given the impression, as interpreted by Mr. Ewald, that the SAF stilling basin is economically the best for all conditions. If he has, he wishes emphatically to correct that impression. There are many locations where other types of outlet structure should be used. The writer is confident, however, that the SAF stilling basin can compete with alternate types of outlet which might be used in similar locations.

Bibliography.—

(11) **"Open Channel Flow at High Velocities,"** by L. Standish Hall, *Transactions*, ASCE, Vol. 108, 1943, p. 1394. (*a*) p. 1448.

(15) **"The Hydraulic Jump in Terms of Dynamic Similarity,"** by B. A. Bakhmeteff and A. E. Matzke, *ibid.*, Vol. 101, 1936, p. 630. (*a*) p. 676. (*b*) p. 643. (*c*) p. 660. (*d*) 0. 643, Fig. 6, curve L/d_2. (*e*) p. 638, Fig. 3, ϵ_j. (*f*) p. 643, Fig. 6.

(34) **"Baffle-Pier Experiments on Models of Pit River Dams,"** by I. C. Steele and R. A. Monroe, *ibid.*, Vol. 93, 1929, pp. 451–546. (*a*) p. 518. (*b*) p. 499.

(38) **"Flow of Water in Steep Chutes with Special Reference to Self-Aeration,"** by R. Ehrenberger, translation by U. S. Bureau of Reclamation, Denver, Colo., 1926.

(41) **"Hydraulic Models,"** *Manual of Engineering Practice No.* 25, ASCE, 1942. (*a*) p. 14, Eq. 40.

(42) **"The High Velocity Flow of Water in a Small Rectangular Channel,"** by Warren W. DeLapp, thesis presented to the University of Minnesota, at Minneapolis, in June, 1947, in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Hydraulic Engineering.

(43) **"Flow through Diverging Open Channel Transitions at Supercritical Velocities,"** by Fred W. Blaisdell, SCS, U.S.D.A., June, 1947 (mimeographed).

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

NEW FORMULAS FOR STRESSES IN CONCRETE PAVEMENTS OF AIRFIELDS

Discussion

BY H. M. WESTERGAARD

H. M. WESTERGAARD,[23] M. ASCE.—The curves presented by Mr. Horonjeff serve the useful purpose of showing the numerical meaning of the formulas when applied to a range of practical cases. These curves give renewed emphasis to the fact that the computed values of the stresses are not very sensitive even to fairly great variations of the modulus k of subgrade reaction. The particular test to determine k mentioned by Mr. Horonjeff deserves attention because it is fairly simple, can be made in advance, and has been found to give usable values of k within some range of applications. Outside this range, for example, in the case of very thick pavements, it is probable that values of k obtained directly from this test will require some coefficient of correction.

The fact is that—as has often been stated—the use of a definite value of k is only an empirical makeshift justified by usable results. The modulus k is not a simple mechanical property of the subgrade. The reaction of the subgrade at any one place depends not only on the deflection at that place but on the general pattern of deflections within the surrounding area; and the value of k that serves best under given circumstances depends on this pattern as well as on the mechanical properties of the subgrade itself. Therefore k becomes a property of the subgrade and the pavement and the character and position of the load. Thus, since the pattern of deflections when the load is at an edge is different from the pattern when the load is in the interior of the area, the proper value of k may well be different for the edge and the interior. These remarks supplement the comments by Mr. Horonjeff and may be of some help in explaining the results presented by Mr. Bone in his discussion.

The experiments with a model reported by Mr. Bone are unquestionably very valuable. It is gratifying that the curve representing the measured strains has the same general form as that obtained from the formulas, but it is puzzling that the measured strains are consistently smaller than the com-

NOTE.—This paper by H. M. Westergaard was published in May, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1947, by Robert Horonjeff; and February, 1948, by Evan P. Bone.

[23] Gordon McKay Prof. of Civ. Eng., Graduate School of Eng., Harvard Univ., Cambridge, Mass.

puted strains. A minor part of this discrepancy may possibly be explained by the finite length of the gage line, but the explanation nearest at hand would seem to be that the value of k should be greater for the edge than the value that gives the best results for the interior of the area. A correction factor between 1 and 2 may be needed to account for the supporting influence of the material outside the edge. To see what this means numerically one may refer to Example 2 and Eq. 27 in the paper. In this example of a 50,000-lb load on a 10-in. pavement an edge stress of 741 lb per sq in. was computed with $k = 200$ lb per cu in. If this k were changed to 400 lb per cu in., the computed stress would be reduced to 626 lb per sq in. This reduction is of the order of magnitude indicated by Mr. Bone's results.

Another possibility involves consideration of the energy of shearing stresses near the edge on vertical sections perpendicular to the edge. In the theory this energy was assumed to be relatively negligible. If it is found not to be negligible, this energy may account for a part of the discrepancy. A study of this influence would be difficult, but might prove to be worth undertaking.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

FORECASTING PRODUCTIVITY OF IRRIGABLE LANDS

Discussion

BY HOWARD N. MAGNESS

HOWARD N. MAGNESS,[19] ESQ.—It is gratifying that the paper by Mr. Muldrow has resulted in such appropriate and beneficial discussion. The writer feels his inability to comment on the discussions and to prepare a closing discussion in the way Mr. Muldrow would have done.[19a] Nonetheless, in view of his professional association and fellowship with Mr. Muldrow, the writer welcomes the opportunity to conclude the discussion.

After many years of experience on irrigation planning, management and accomplishments and observations on changes in land use with specialization of crops and increased production, Mr. Muldrow frequently stated that he believed preliminary forecasts of production and basic economic estimates in support of projects were too conservative. The comments of the several discussers indicate a growing desire for more complete economic consideration of returns from proposed improvements. This tendency is rapidly assuming greater importance, especially in the more costly projects involving major engineering developments that affect large segments of land and population, and can result only in a firmer foundation for future planning.

It is realized that the material used by Mr. Muldrow was not as complete as might be desirable. The comments of Messrs. Lewis and Johnson concerning the use of additional crops are pertinent, as are those of Messrs. Blaney and Van Loo, concerning the discrepancy of yields in the Imperial Valley in California. The point raised by Mr. Fox relative to most desirable temperature base is considered reasonable, and comparison of the effect of the two temperature bases mentioned would be desirable.

The suggestion of Mr. Noble that further investigation be made toward evolving a final cash return per acre is especially helpful. It is true that, in

NOTE.—This paper by the late W. C. Muldrow was published in February, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1947, by Arthur F. Johnson, H. W. Van Loo, and Will H. Noble; June, 1947, by Charles Kirby Fox; October, 1947, by M. R. Lewis; November, 1947, by Harry F. Blaney; and December, 1947, by R. L. Parshall.

[19] Engr. (Soils), Portland Dist., Corps of Engrs., Dept. of the Army, Portland, Ore.

[19a] Mr. Muldrow died on January 14, 1947.

general, crops produced in areas of high temperatures have as high a value—or perhaps an even higher value—per pound or other unit of production, as those grown in cooler areas. For this reason, where the temperatures exceed the optimum for the growth of potatoes or alfalfa, they should be replaced by other crops adapted to these temperatures, which possibly have an even higher yield value per acre.

It is realized that the basic paper and the discussions concerning it are too brief to present, fully, even this one feature of irrigation planning—that is, considerations of temperature in relation to crop production. Although vital, temperature is still only one of the factors affecting a project. Mr. Muldrow realized that his paper was only introductory to a complex and very interesting point of investigation. Further study and development of the subject would be most desirable.

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DISCUSSIONS

UNDERGROUND CONDUITS—AN APPRAISAL OF MODERN RESEARCH

Discussion

BY M. G. SPANGLER

M. G. SPANGLER,[54] M. ASCE.—The time and effort put forth in preparing this paper have been more than compensated for by the stimulating ideas which have been expressed by the various discussers. Of particular value are the data presented by Mr. Binger relative to the pressures measured on a concrete box culvert under a 50-ft fill on the Panama Railroad. The close correlation between the measured pressures and those indicated by the Marston theory when applied to the conditions of the installation serve, powerfully, to reinforce the confidence which engineers may place in the principles involved in the theoretical development.

Mr. Binger's data are significant for two reasons: First, they show the load produced by a fill two and one-half times higher than any previous fill for which loads have been measured; and, second, the fill was compacted by modern heavy equipment during construction.

Mr. Feld is entirely justified in referring to the researches on underground conduit loads which were conducted at the University of North Carolina in Chapel Hill. In preparing any paper the matter of scope becomes highly important and in this case it was decided to limit the paper to the work which had been done at Iowa State College in Ames. Dean Marston has made an exhaustive study of the reports covering the North Carolina experiments and has concluded that the measured loads in those tests conform closely to the results calculated by the Marston theory, as he has stated in his discussion.

It is difficult to subscribe to Mr. Feld's characterization of the problems discussed in the paper as purely academic. Although it is true, as everyone knows, that methods of earth handling and construction have undergone radical changes during the past quarter of a century, the principles of stress transfer embodied in the Marston theory are ageless and will be applicable many years

NOTE.—This paper by M. G. Spangler was published in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1947, by Wilson V. Binger, and Jacob Feld; February, 1948, by Zdeněk Bažant, Jr., Anson Marston, and George E. Shafer; and March, 1948, by E. F. Kelley, and Bailey Tremper.

[54] Research Prof., Civ. Eng. Dept., Iowa Eng. Experiment Station, Iowa State College, Ames, Iowa.

hence when present-day construction methods become outmoded. The ditch conduit analysis may be applied with equal validity to a hand-dug or a machine-dug trench without reference to the slope or irregularity of the sides, provided the correct ditch-width factor is used in the analysis. If the backfill material is placed in the condition which Mr. Feld describes as "better" than relatively loose earth, considerable harm may result in that the loads on the pipe may be very greatly increased as compared to the loads under the more usual manner of backfilling. In the example cited where the backfills of drainage trenches became relatively hard lines in the subgrade, there is little doubt in the writer's mind but that the load at the base of the trench was relatively high, because the transferred shearing stresses would be additive to the weight of the backfill instead of subtractive as in the more usual case.

The method of constructing projecting conduits in trenches dug through compacted embankments has distinct advantages from the standpoint of loads produced on the structure, in addition to the advantages mentioned by Mr. Feld, since placing the pipes in ditches tends to cause the shear stress increments to act upward on the prism of material in the ditch, provided the backfill is loosely placed. The California Highway Department has developed methods of construction based on this principle which appear to be efficient in reducing loads on the pipes. These procedures, together with excellent bedding and backfilling practices, have permitted a substantial increase in safe heights of fill over drainage conduits under highways in that state. However, it seems doubtful whether the practice of building an embankment first and then trenching through it to construct a drainage line can ever be widely applicable, inasmuch as climatic conditions in many localities are such that a heavy rainstorm between the time of completion of the embankment and the construction of the trench through it might be very detrimental or even disastrous. The favorable load condition produced by this method of construction is recognized, however, and it is clearly within the scope of the Marston load theory. The deeper the trench through the embankment in relation to the trench width, and the more loosely the backfill is placed in the region immediately above the conduit, the farther toward the left side of Fig. 8 the situation falls and the less will be the load on the structure.

Mr. Tremper's remarks indicate that similarly good results are being obtained in the State of Washington by installing pipe culverts in sidehill ditches to one side of the watercourse. Where practicable, this procedure would appear to be better than that suggested by Mr. Feld, because drainage water could be cared for in the natural watercourse during construction of the culvert. The discussions by both Messrs. Feld and Tremper lend emphasis to the more favorable load situation that can be achieved in culvert construction when specific attention is given to creating conditions in which pressures are transferred by shear from the interior prism of soil to the exterior masses.

From Prague, Czechoslovakia, comes a very interesting analysis of concrete pipe rings from which values of the load factor for cradled pipe have been determined. There is no difference of opinion between Professor Bažant and the writer relative to the analytical determination of bending moments in a reinforced concrete pipe. This procedure is precisely that employed by the

writer prior to 1933 which led to the development of Eq. 14*b*. The difficulties in design arise when one attempts to convert the calculated moments into stresses in the steel and concrete of which the pipes are made.

Professor Bažant's calculated load factors shown in Table 1 are approximately 10% greater than the average experimental values obtained by W. J. Schlick, M. ASCE, and James W. Johnson.[55] These differences are not significant and readily may be accounted for by the fact that the sand-bearing load was assumed to be uniformly distributed in the analysis, whereas it is known that the pressure in the test is greater at the center of the bearing than at the sides and that it approaches a parabolic distribution.

However, several circumstances and assumptions connected with the analysis contained in the discussion lead to the conclusion that the correlation shown in Table 1 may be more coincidental than basic. For example, it is assumed that: "The maximum moment occurs at points a and b, where the first cracks occur * * *." In the Schlick and Johnson experiments, 94% of the cradled specimens cracked first at the top of the pipe. The first cracks in the remaining 6% occurred simultaneously either at the top and bottom or at the top and sides of the pipe. Also, in every case the cradle cracked in the region below the bottom crack in the pipe and at the same time that the bottom crack occurred. In no case did the first cracks occur at the junction of the pipe wall and the cradle as assumed by Professor Bažant.

The comparison between computed and experimental values of the load factor for ultimate loads is probably not valid, even though the values appear to be reasonably close numerically, because at ultimate load the pipe and the cradle have long since ceased to be a continuous arch body. At ultimate loadings, the experimental pipes were cracked in many places, the cradles were broken into two parts, and the bond between the cradles and the pipes was destroyed, so that the structures were considerably different after they were cracked from the structures which Professor Bažant analyzed. For these reasons it would seem that a comparison between calculated and observed load factors at "first crack" load would be much more appropriate. Such comparisons for the three cradle types included in Table 1 show that the calculated load factors exceed the observed values by 18%, 37%, and 35%, respectively.

Mr. Shafer's discussion is a most welcome addition to the literature on the supporting strength of flexible pipe conduits. He has had wide experience in this field, has made extensive observations of the performance of this type of structure, and has brought rare good judgment to the interpretation of his observations. The writer has a very high regard for the utility of Mr. Shafer's empirical Eq. 26. It has served both the industry and the users of metal culvert pipes very well since its inception in 1926.

In the writer's opinion, however, the flexible pipe problem is far too complex to rely solely on experience for a solution. Theory is needed to supplement judgment and to guide the interpretation of the shotgun pattern of the facts gleaned in the field. The empirical formula falls short in the achievement of scientific design of flexible pipe culverts and tends to stifle progress in that

[55] "Concrete Cradles for Large Pipe Conduits," by W. J. Schlick and James W. Johnson, *Bulletin No. 80*, Iowa Eng. Experiment Station, Ames, Iowa, 1926, p. 39.

direction, since it does not take into account several important variables that influence the problem. One of these is the passive resistance of the soil side fills, which is widely recognized as a highly important factor (probably the most important single factor) on which pipe deflection depends. The empirical formula throws all the effect of side pressure into the factor k, and the best that can be expected under these circumstances is the determination of "average" deflection, which may be greatly exceeded in individual cases. Very few field data are available to the profession from which to judge the correlation between actual deflections developed in the field and deflections predicted by the empirical formula. Such data as the writer has seen show a very wide range of deflections, many individual specimens being in excess of 100% above the "average," and extreme cases being from 400% to 500% above.

Another variable factor which the empirical formula submerges in the "average" is the relationship between the height of fill and the load on a culvert. It is unfortunate, but nonetheless true, that the load and the height of fill are not related by a constant factor for all culverts. This fact is clearly shown by Fig. 8. In the present state of knowledge it is difficult to predict values of the settlement ratio, which has such an important effect on the relationship between load and height of fill. Nevertheless, it is present and effective in every culvert installation and scientific progress demands that it be recognized and studied. This same comment applies to the projection ratio, except that the latter factor is more easily determined for a specific pipe installation.

For these reasons and others, the writer does not consider that the fill height data given in Table 2 are validly comparable. It would be the purest coincidence if the fill heights shown in the parallel columns were in agreement. The only valid check on Eq. 15 is a comparison of the deflections computed by the formula with actual measured values when all the factors relative to a culvert installation which influence the deflections are known or can be estimated with a reasonable degree of precision.

When Eq. 15 was developed and published, it was intended to be used to compute the deflection of a flexible pipe culvert when the diameter of the pipe and its physical properties (the height of fill, the settlement ratio, the projection ratio, the width of bedding, the unit weight of the fill, and the character of the soil as it influences passive resistance pressures) are known. Immediately after publication, many engineers turned the formula around and attempted to use it to calculate a fill height that would produce some predetermined deflection of the pipe. This practice has led to some difficulties and erroneous concepts concerning the formula. Mr. Kelley rejects the formula as giving unreasonable results, because the fill height to produce a given deflection, expressed as a constant percentage of pipe diameter, is larger for large pipe than it is for some smaller pipe sizes. His point would be well taken if it could be definitely stated that, when two pipes of different diameters have deflected under a fill an amount equal to 5% of their diameters, they have both progressed the same distance toward failure and are, therefore, on a comparable basis as far as strength is concerned. The writer does not believe this is true.

This premise opens the whole question as to the theory of failure of flexible pipes and a suitable criterion for an allowable design limit. The metal pipe industry has for many years suggested a deflection limit of 5% of the diameter as being a suitable criterion and many engineers have accepted this value. The writer has looked on the 5% limit as a good value to "shoot at," but he has come to realize that neither it nor any other constant percentage-of-diameter deflection represents the same strength situation for all diameters of pipes. As evidence on this point, the writer has investigated the pipes for which Fig. 28 was drawn by computing the fill heights to produce a constant value of bending moment at the bottom of the pipes. The equation for the moment at the bottom of a flexible pipe under field loading conditions (for a bedding angle of 35°) is[56]

$$M = 0.176\ W_c\, r - 0.166\ h\, r^2 \quad \text{(33a)}$$

in which

$$h = \epsilon\, \frac{\Delta}{2} \quad \text{(33b)}$$

Substituting Eq. 15 in Eq. 33*a* and using the same values of various factors that were used by Mr. Kelley, it is possible to compute heights of fill over the various diameter pipes to produce the same bending moment in each of the pipes. The bending moment in the 36-in. pipe of Fig. 28 was arbitrarily chosen as the common value for all the pipes. The results are shown in Fig. 29. On the basis of this criterion the height of fill decreases with increasing pipe diameter, which seems to be more reasonable. Since both Eq. 15 and Eq. 33 are based on the elastic analysis, the author believes that Fig. 28 throws a cloud of uncertainty on the validity of applying a constant percentage-of-diameter deflection limit to all sizes of pipe, rather than on the validity of Eq. 15.

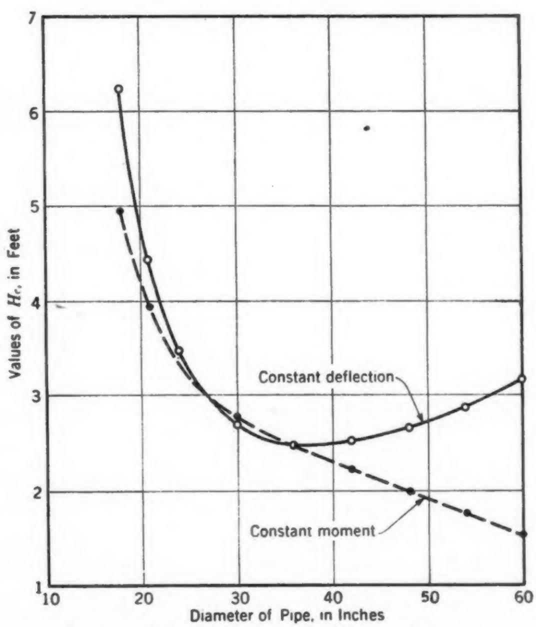

FIG. 29.—HEIGHT OF FILL VERSUS PIPE DIAMETER

Mr. Shafer goes further and applies the 5% limit to pipes loaded in two-edge bearings and compares the loads to produce this deflection by elastic analysis and by actual test. Since the smaller sized pipes tested were obviously

[56] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Iowa Eng. Experiment Station, Ames, Iowa, 1941, p. 58.

loaded beyond the elastic limit of the metal, the calculated loads to produce 5% deflection were considerably greater than the test loads. This discrepancy between calculated loads and test loads is cited by Mr. Shafer as an indication of a weakness in Eq. 15. The writer believes that the comparisons in Table 3 are wholly fortuitous and irrelevant and that Mr. Shafer's conclusions based on them are not valid.

In support of this contention, it is pointed out that, if the test specimens had been made of metal having a higher elastic limit, the difference between the loads calculated by Eq. 27 and the test loads would have been materially less. In Fig. 23, the metal in the 36-in. pipe for which load-deflection data are plotted apparently reached its elastic limit at a load of 1,056 lb per lin ft, or 88 lb per lin in. The bending moment at the top and bottom of a circular ring loaded in two-edge bearing is[57]

$$M = 0.318\, P\, r \qquad (34)$$

in which P is the load per unit length of pipe; and r is the radius of the pipe. By the flexure formula,

$$s = \frac{M\, c}{I} \qquad (35)$$

The elastic limit of the metal in this pipe is approximately 46,000 lb per sq in. by Eqs. 34 and 35.

Corrugated metal pipe culverts are made of many different kinds of iron and steel, ranging from commercially pure iron having a relatively low elastic limit to copper and copper-molybdenum iron or steel alloys having high elastic limits. In the research work on which Eq. 15 was based, the metal pipes furnished for the tests by four different manufacturers varied in elastic limits from 47,800 lb per sq in. to 65,500 lb per sq in. If the 36-in. pipe of Fig. 23 had been made of metal having an elastic limit of 66,000 lb per sq in. instead of 46,000 lb per sq in., the load-deflection diagram would have been approximately as shown by the dotted line in Fig. 30 and the difference between actual loads and those computed by Eq. 27 would have been much less.

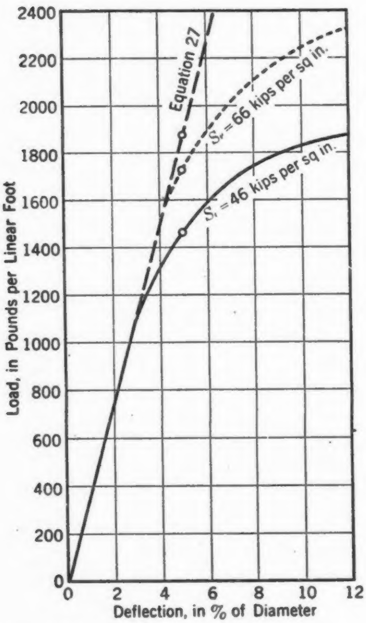

FIG. 30.—LOAD-DEFLECTION DIAGRAM FOR CORRUGATED METAL PIPE CULVERTS

Likewise, the data shown in Table 3 are intimately dependent on the elastic limits of the particular pipes tested. It is the writer's view that these data do not provide a proper basis for generalization without some detailed knowledge of the elastic limits of the metals from which the test specimens were made.

[57] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Iowa Eng. Experiment Station, Ames, Iowa, 1941, p. 14.

The data in Table 3 are irrelevant because the load and reaction in a two-edge bearing test are much more concentrated than in the field loading for which Eq. 15 was developed and it seems probable that the difference between computed loads and actual loads would be much less in the latter case, even when the pipe metal is stressed beyond its elastic limit at localized points—that is, the load-deflection curve under field loading would probably deviate from a straight-line elastic relationship much less rapidly than under concentrated loading. Mr. Shafer appears to agree with this view. Obviously, also, pipes made of metal having a high elastic limit would deviate less from the elastic line and would have less deflection for a given load than would pipes made from low elastic limit metal.

The data shown in Table 3 again raise the question of whether it is valid to apply a constant limiting deflection to all sizes of pipe. Mr. Shafer has given an interesting account of the origin of the commonly accepted 5% limit. He would do the profession a service if he would publish the data on which the values were established. It is pertinent to ask whether the "average safe maximum deflection" which was observed was the same percentage value for all pipe diameters.

Mr. Shafer has raised some appropriate questions concerning the nature of the modulus of passive resistance of soils, and the writer agrees that there is much yet to be learned about this property and its application to the flexible pipe problem. It is hoped that future research, both in the laboratory and in the field, will shed further light on this factor concerning which so little factual data exist. In particular, information is needed to determine more definitely the relationship between passive pressures and movement of the sides of the pipe, the distribution of the passive pressures, the effect of fill height and pipe rigidity on the passive pressure modulus, and soil test procedures on which to base estimates of the value of the modulus for design use. Such added information is needed in order to refine and improve Eq. 15 and to develop a parallel formula for strutted flexible pipe culverts.

The author is deeply indebted to those who have discussed the paper and wishes to express to them his personal thanks. He believes the discussions have added materially to the understanding of the underground conduit problem.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF FRAMES WITH ELASTIC JOINTS

Discussion

By L. E. Grinter

L. E. Grinter,[8] M. ASCE.—The statement of the author (see "Introduction") that the "application of end moment distribution * * * to the analysis of structures with semi-rigid joints has resulted in computations which * * * are complex and tedious" must be considered in relation to the author's preferred procedure. There is no such thing as complexity or simplicity in structural analysis except as relative terms. A procedure[9] of alternately distributing moments and of balancing end angle changes, which requires no explanation for those who use these tools, is at least equally convenient as a method of solving the problems discussed in this paper. All ratios such as stiffness and the carry-over factors remain unchanged. The method has the added advantage that it may be used where the moment-rotation curve for the joint represents nonlinear elasticity. The distribution procedure again shows its natural advantage of keeping the designer closely in contact with the physical action of the structure as it deforms.

Note.—This paper by Ralph W. Stewart was published in December, 1947, *Proceedings*.

[8] Research Prof. of Civ. Eng. and Mechanics, Illinois Inst. of Technology, Chicago, Ill.

[9] "Theory of Modern Steel Structures," by L. E. Grinter, Macmillan Co., 1937, Vol. 2, p. 262.

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DISCUSSIONS

INFLUENCE LINES FOR CONTINUOUS STRUCTURES BY GEOMETRICAL COMPUTATION

Discussion

BY RALPH W. STEWART, YI-MAI YAO, AND OTTO GOTTSCHALK

RALPH W. STEWART,[10] M. ASCE.—The various examples demonstrated by the author all relate to the influence line for some one specific point, or to the influence lines for supports only.

In the design of reinforced concrete rigid frame bridges, it is desirable and entirely practicable to construct the envelope that passes through the maximum influence line ordinates for all points of the structure. This will enable the computer to plan the reinforcing bars so that they will conform accurately to the requirement at each point.

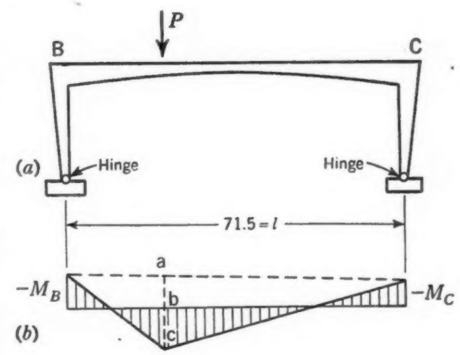


FIG. 13

Fig. 13(*a*) indicates a single-span rigid frame bridge. Modern design specifications require the decks of such bridges to be designed as if the bases were hinged.[11]

The constants of flexure for the deck and abutments are determined and a traverse constructed[12] as shown in Fig. 14(*a*). This shows the effect of a fixed-end moment of 7.28 applied at the left end only of the deck with sidesway prevented. To determine the properties of the structure for any symmetrical loading simply reverse Fig. 14(*a*) on itself and combine the moments. The result, Fig. 14(*b*), shows the haunch moments (5.69) produced by sym-

NOTE.—This paper by Dean F. Peterson, Jr., was published in September, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1947, by Ralph E. Spaulding.

[10] (*Retired*), Los Angeles, Calif.

[11] "Specifications for Highway Bridges," Am. Assn. of State Highway Officials, Washington, D. C., 1944.

[12] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, ASCE, Vol. 104, 1939, p. 527, Fig. 4.

metrical fixed-end moments of 7.28. To determine the effect of a single fixed-end moment corrected for sidesway, divide the haunch moments in Fig. 14(*b*) by 2 and write them in Fig. 14(*d*); also, write the fixed-end moment 7.28 at the left end of the deck and Fig. 14(*d*) will then show the effect of this fixed-end moment with sidesway permitted.

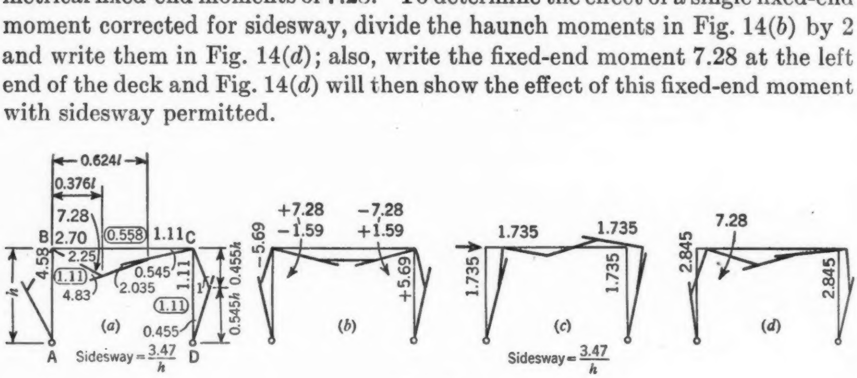


FIG. 14

A more easily understood but longer method of correcting for sidesway is to note that the sidesway force in Fig. 14(*a*) is $\frac{3.47}{h}$. Since the structure is symmetrical, one half of this goes to each abutment, thus yielding Fig. 14(*c*). Now combine Fig. 14(*a*) with Fig. 14(*c*), and Fig. 14(*d*) is again obtained.

Figs. 14(*b*), 14(*c*), and 14(*d*) now cover all conditions of flexure. Fig. 14(*a*) is simply a means to this end and can be abandoned thereafter. For this very simple structure Fig. 14(*a*) need not have been constructed. A traverse for symmetrical loading, similar to Fig. 14(*b*), can be constructed directly by two slide rule settings. A figure similar to Fig. 14(*d*), will then, as already shown, follow directly from it. For more general cases, however, a diagram corresponding to Fig. 14(*a*) must be drawn.

TABLE 8.—FIXED-END MOMENT COEFFICIENTS FOR CONCENTRATED LOAD (FIG. 13)

Load of unity at point[a]	COEFFICIENTS FOR: (FEM)$_L$ (1)	(FEM)$_R$ (2)	Col. 1 plus Col. 2 (3)	$M_{BC} = M_{BA}$ (4)	Simple beam moment[b] (5)	Moment under load[b] (6)
1	0.0744	0.0053	0.0797	2.22	5.46	3.24
2	0.1290	0.0210	0.1500	4.19	9.94	5.75
3	0.1628	0.0463	0.2091	5.84	13.40	7.56
4	0.1750	0.0785	0.2535	7.08	15.90	8.82
5	0.1679	0.1136	0.2815	7.85	17.38	9.53
6	0.1455	0.1455	0.2910	8.12	17.88	9.76

[a] Point 6 is the center line of the span. [b] See Fig. 13(*b*): The moment under the load (bc) equals the simple beam moment (ac) minus ab.

Cols. 1 and 2, Table 8, show the fixed-end moment coefficients by which ($P\,l$)-values are multiplied to compute the fixed-end moments for concentrated loads at each of the $\frac{1}{12}$ length points along the span. There are several available tables and charts giving these coefficients. With structural symmetry

and hinged bases the haunch moments of this frame will always be equalized by the sidesway; therefore, the computer can combine the items in each line of Cols. 1 and 2 and obtain Col. 3.

Each haunch moment for a unit load recorded in Col. 4, Table 8, will then be $\frac{2.845}{7.28}$ (Fig. 14(*d*)) multiplied by the adjoining item in Col. 3 and the length of the deck member. The simple moments shown in Col. 5 are then computed. By deducting the moments in Col. 4 from the simple moments the moments under the load are obtained as shown in Col. 6, Table 8. Fig. 13(*b*) illustrates this computation for the general condition. The ordinate ac will be recognized as the maximum simple beam moment and the ordinate bc of the shaded area ($\overline{ac} - \overline{ab}$) is the moment under the load. For symmetrical structures with hinged bases it will not be necessary to interpolate to obtain the ordinate ab as M_B and M_C are equal for all positions of the load.

TABLE 9.—INFLUENCE ORDINATES FOR A UNIT CONCENTRATED LOAD[a]

Line	0	1	2	3	4	5	6
1	−2.22	+3.24	+2.73	+2.23	+1.72	+1.22	+0.71
2	−4.19	+0.78	+5.75	+4.76	+3.77	+2.79	+1.78
3	−5.84	−1.37	+3.09	+7.56	+5.07	+4.58	+3.09
4	−7.08	−3.10	+0.87	+4.85	+8.82	+6.33	+4.84
5	−7.85	−4.38	−0.90	+2.58	+6.05	+9.53	+7.05
6	−8.12	−5.14	−2.16	+0.82	+3.80	+6.78	+9.76
7	−7.85	−5.87	−2.89	−0.40	+2.08	+4.57	+7.05
8	−7.08	−5.09	−3.11	−1.12	+0.87	+2.85	+4.84
9	−5.84	−4.35	−2.86	−1.37	+0.11	+1.60	+3.09
10	−4.19	−3.19	−2.20	−1.20	−0.21	+0.78	+1.78
11	−2.22	−1.73	−1.24	−0.75	−0.26	+0.22	+0.71
12	−62.48	−33.72	−15.36	−4.84	−0.47	....	....
13	....	+4.02	+12.44	+22.80	+32.29	+41.25	+44.70

[a] Point 0 is at end B, Fig. 13(*a*), and the same values, by symmetry, apply to end C (point 12).

To construct Table 9 first write in the values under point 0 which show the moments at ends B and C. These are taken from Col. 4, Table 8. The underscored items running diagonally across the upper half of Table 9 are the moments under the load taken from Col. 6, Table 8. Each is written where its line number intersects its column number. All other items in Table 9 are computed by horizontal interpolation. A property of this tabulation for a symmetrical frame is that it is symmetrical in all directions about the point where line 6 (Table 9) intersects the last column, point 6; in other words, about the center of the structure. Therefore, Table 9 may be constructed for only half the length of the deck as shown. The vertical columns can now be plotted and will give the influence lines for a unit load shown in Fig. 15.

The summation of the positive and negative influence line ordinates in Table 9 serves two purposes—(1) placing discontinuous sections of uniform live load on the span to get maximum moments, and (2) checking the accuracy of the table. The check is as follows: The simple moment for a load of 1 kip per ft on the 71.5-ft span is $(71.5)^2$ = 638 ft-kips. From the first column of

Table 9 (point 0, line 12), $M_B = 62.48 \times \frac{71.5}{12} = 373$. Subtracting M_B from the simple moment gives 265 ft-kips as the moment at the center of the span. From the last column of Table 9 (point 6, line 13) the moment at the center

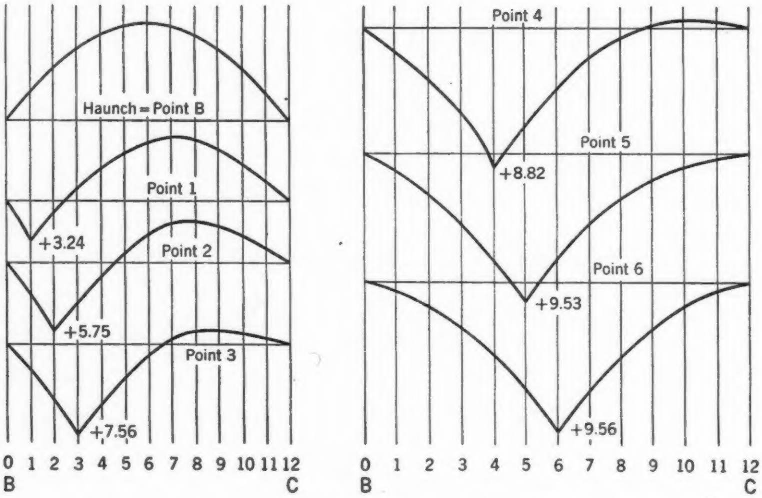


FIG. 15

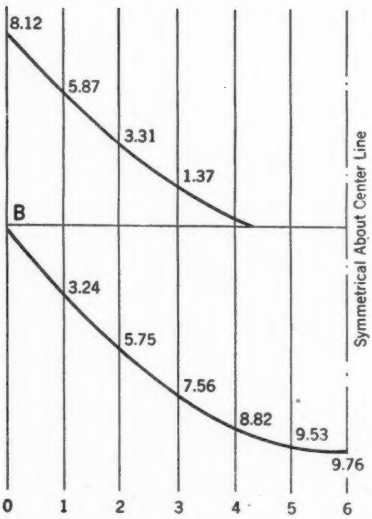


FIG. 16

of the span is $44.7 \times \frac{71.5}{12} = 266$ ft-kips, which checks.

After this check is accomplished Fig. 16 may be drawn. This shows maximum positive and maximum negative moments all along the span due to a concentrated moving load of unity. From Table 9 or the influence line graphs, the moments caused by any live load are readily computed and combined with the dead load, temperature, and other moments so that the required reinforcement for every cross section along the member can be determined. It is not necessary to plot the influence lines (Fig. 15) except to detect errors in the computations which will be disclosed if the points on the curves are obviously out of line.

Many engineers and writers whose opinions carry weight have stated that the computation of influence lines is seldom desirable and that the time and expense of such computations can be saved.

For a symmetrical, single-span structure only two computation sheets are needed for a complete influence line computation covering the bending moments. If a symmetrical structure has three spans with the outer ends simply supported (which is a very common form of reinforced concrete grade-separation structure), three computation sheets will be required. The time or expense of this work is not excessive if the computer is familiar with it. The author should be commended for writing a paper which will tend to increase the supply of such computers.

This discussion supplements the author's presentation by extending the computations to yield the maximum moment at every point in a structure instead of at specified points.

Yi-Mai Yao,[13] Esq.—The author has given a method of plotting influence lines for continuous structures by computing the elastic curve due to a unit displacement corresponding to the "action" being studied, and the unit displacement being located at the section under consideration. The formula for computing the ordinates of an elastic curve, such as Eq. 10, involves the joint rotation θ. The θ-value can be obtained directly and quickly by balancing the angle changes[14] produced by the "unit displacement," a method presented by L. E. Grinter, M. ASCE.

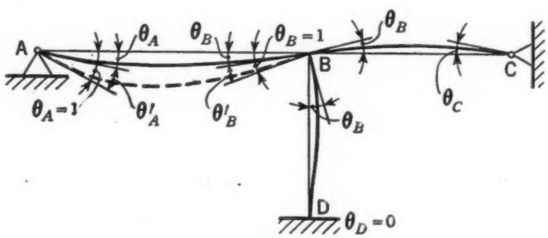


Fig. 17

According to equations derived by Professor Grinter[15] the modified stiffness κ_{ABm} and the modified carry-over factor γ_{ABm} for the direct distribution process are as follows:

$$\kappa_{ABm} = \frac{4\,\kappa_{AB}}{3 + \dfrac{\kappa_{BA}}{\Sigma\kappa'_{Bm}}} \qquad (23)$$

and

$$\gamma_{ABm} = -\frac{2\,\dfrac{\kappa_{AB}}{\Sigma\kappa'_{Bm}}}{3 + \dfrac{\kappa_{BA}}{\Sigma\kappa'_{Bm}}} \qquad (24)$$

in which $\Sigma\kappa'_{Bm}$ indicates the summation of the modifier stiffness of the member tributary to joint B except BA plus the unmodified stiffness of BA.

[13] Junior Engr., TangKu New Harbor Construction Bureau, TangKu, Tientsin, China.

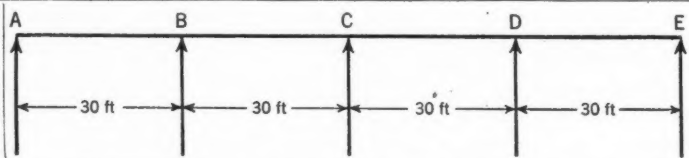
[14] "Analysis of Continuous Frames by Balancing Angle Changes," by L. E. Grinter, *Transactions*, ASCE, Vol. 102, 1937, pp. 1020–1067.

[15] *Ibid.*, p. 1035, Eqs. 6 and 7.

TABLE 10.—Modified Solution of Example 9

(a) Distribution

Symbol:

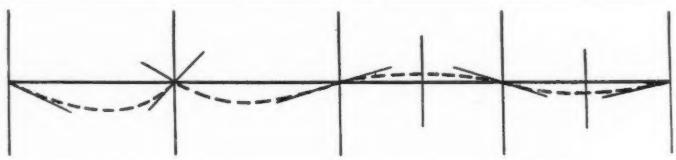


Line No.	Symbol	A	B		C		D		E
1	Joint	A	B		C		D		E
2	Member	AB	BA	BC	CB	CD	DC	DE	ED
3	κ..........	2.00	2.00	4.00	4.00	2.00	2.00	1.00	1.00
4	κ_m..........		2.00	4.39	4.36	2.18	2.41	1.00	
5	$\kappa_m/\Sigma\kappa_m$.....	1.000	0.313	0.687	0.667	0.333	0.706	0.294	1.000
6	γ_m	−0.188	−0.500	−0.355	−0.363	−0.363	−0.189	−0.500	−0.178

(b) Influence Line for Moment M_B

Line No.	Symbol	AB	BA	BC	CB	CD	DC	DE	ED
7	θ_1.........			+1.00					
8	θ.........	−0.34	+0.69	−0.31	+0.11	+0.11	−0.04	−0.04	+0.02

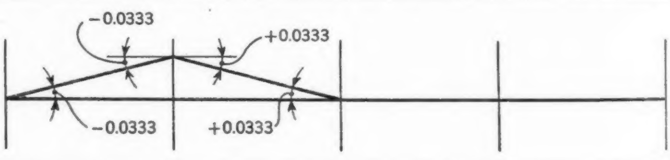
(b)



(c) Distribution of Angle Changes

Line No.	Symbol	AB	BA	BC	CB	CD	DC	DE	ED
9	θ_1	−0.0333(1)	−0.0333(2)	+0.0333(3)	+0.0333(4)				
10	Bal. θ(AB) ..	−0.0333	0	0	0	0	0	0	0
11	Bal. θ(BA) ..	−0.0115	−0.0104	−0.0104	+0.0037	+0.0037	−0.0014	−0.0014	+0.0007
12	Bal. θ(BC) ..	−0.0115	+0.0229	+0.0229	+0.0037	+0.0037	−0.0014	−0.0014	+0.0007
13	Bal. θ(CB) ..	−0.0020	+0.0040	+0.0040	+0.0222	+0.0222	−0.0080	−0.0080	+0.0040

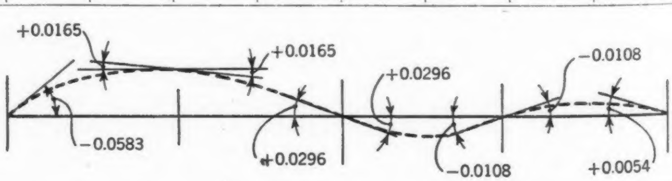
(c)



(d) Influence Line for Reaction, R_B

Line No.	Symbol	AB	BA	BC	CB	CD	DC	DE	ED
14	θ..........	−0.0583	+0.0165	+0.0165	+0.0296	+0.0296	−0.0108	−0.0108	+0.0054

(d)



The simply supported continuous girder in Example 9 can be solved easily by direct distribution, as shown in Table 10, for plotting the influence lines for moment M_B and reaction R_B.

Line 3, Table 10, shows the unmodified stiffness of member, κ. Line 4 shows the modified stiffness κ_m by Eq. 23; for example, $\kappa_{CBm} = \dfrac{4 \times 4}{3 + \dfrac{4}{4+2}} = 4.36$ and $\kappa_{DCm} = \dfrac{4 \times 2}{3 + \dfrac{2}{2 + 4.36}} = 2.41$. Line 5 shows the $\dfrac{\kappa_m}{\Sigma\kappa_m}$-value, and line 6 shows the modified carry-over factor γ_m; for example, $\gamma_{CBm} = -\dfrac{2\dfrac{4}{4+2}}{3 + \dfrac{4}{4+2}}$ $= -0.363$ and $\gamma_{DCm} = -\dfrac{2\dfrac{2}{2+4.36}}{3 + \dfrac{2}{2+4.36}} = -0.187$. Line 7, Table 10, shows the "unit displacement" corresponding to the influence line for M_B; line 8 shows the direct distribution—for example, $\theta_{BA} = +1.00 \times 0.687 = +0.69$, which, carried over, yields $+0.69 \times -0.500 = -0.34$ to joint A. The rotation, $\theta_1 = 1.00$, must remain at joint B, and therefore, in line 6, -0.31 must remain in member BC. Consequently, $-0.31 \times -0.355 = +0.11$ is carried to joint C; $+0.11 \times -0.363 = -0.04$ is carried to joint D; and $-0.04 \times -0.500 = +0.02$ is carried to joint E. The influence line for M_B can be plotted with θ-values, -0.34, $+0.69$, -0.31, $+0.11$, -0.04, and $+0.02$, as shown in Table 10(*b*). Line 9, Table 10, shows the angle changes $-\theta = 1/30 = 0.0333$ due to a "unit displacement" of support B when joints A, B, and C are hinged (see Table 10(*c*)); line 10 shows the results of balancing the angle change -0.0333 at AB, which member keeps the entire amount since joint A remains hinged. Line 11 shows the results of balancing the quantity -0.0333 at BA. The end BA keeps $-0.0333 \times 0.313 = -0.0104$. The quantities carried over are, for example, $-0.500\ [-0.0104 - (-0.0333)]$ $= -0.0115$ from BA to AB; $-0.335 \times -0.0104 = +0.0037$ from BC to CB; and $-0.363 \times +0.0037 = -0.0014$ from CD to DC. Lines 12 and 13, Table 10(*c*), show the balancing process of angle changes $+0.0333$ at BC and CB. Line 14, Table 10(*d*), shows the final joint rotation θ, the sum of lines 10 to 13 in Table 10(*c*). The influence line for R_B can be plotted by means of Eq. 10 using the unit rise at support B, and the joint rotations listed in the various columns of Table 10(*d*).

The theory supporting Professor Peterson's method is not new; it is merely the modified form of the Müller-Breslau principle. The fundamental Eq. 10 can be derived by the classical moment-area method without any knowledge of "geometry." The author gives a rapid and accurate procedure for computation. Although so many different methods of statical analysis can be used, the writer has considered Professor Grinter's method to be the best, especially for the prismatic member, which avoids the tedious moment computation. Its utilization will increase the value of the author's paper.

OTTO GOTTSCHALK,[16] ESQ.—The neat arrangement of this paper, conforming with current practice, is commendable. The thing that the writer would like to see corrected is the abstract symbolism used in most of the recent papers on structural design, including this one. At an alarming rate, this field is accumulating barriers of learned thought instead of simple methods based on elemental geometrical analysis. Although the author implies that geometrical methods are used, he applies abstract symbols of geometrical units as loads, which is not the same.

A characteristic of the geometrical method of analysis is that the several steps are visible and, thereby, they are an aid to a designer's intuition. Abscissas, ordinates, radii, curvatures, linear or angular displacements or deformations, and the separation or the breaking of originally continuous parts are all visible in a truly geometrical method of analysis. Stresses and stress functions and moment and other stress areas are not visible and therefore are not used.

For purposes of analysis, rigid structures are geometrical bodies and must be analyzed by geometrical methods. Every step away from this principle leads to complications. If a given structure is said to be rigid, any geometrical disturbance at one end of one of its members is resisted by the stiffness of adjacent members, so that a unit geometrical displacement at any given section causes deformation all over the structure, bending the originally straight members into curves which are the influence lines. (Generally, the designer is not interested in the true shape of the curve at every point, but in the value of selected ordinates.)

The cross sections at either end of every straight beam or column are perpendicular to the axis before loading. When the structure is deformed these sections are either rotated or translated, or both. When such beams are prismatic, they can be analyzed easily by purely geometrical methods because their curvature changes in proportion to their abscissas, so that, graphically, if the curvature is represented by the ordinate, the curvature at any point is represented by the straight line[17] that joins the ordinates representing curvature at the ends. In true geometrical analysis loads are considered external to the structure. They are not a part of it and do not enter into the analysis. They are only factors with which to multiply ordinate values determined by the geometry of the problem.

By applying equations of influence lines such as those developed by the writer (which the author has cited[3]), the laborious work of computing the tables required by the author is made unnecessary. For prismatic members (span AB, Fig. 11) the equation of the influence line[3] is

$$y = \frac{x\,x'\,(x\,\theta' + \theta\,x')}{l^2} \qquad (25)$$

in which, conforming to the notation of the paper, l is the span length AB; x and x' are the abscissas; and θ and θ' are the rotations of the points A and B,

[16] Engr., Central Argentine Ry., Villa Ballester, Argentina.

[17] "Calculo Estático de Estructuras Rigidas sobre Bases Naturales," by Otto Gottschalk, Librería Hachette, Buenos Aires, Argentina, p. 14, Fig. 13.

[3] *Transactions*, ASCE, Vol. 103, 1938, p. 1019.

respectively. As the author indicates (see "Synopsis"), the methods in current use require the computation of a considerable number of points to obtain an influence line. By contrast, the geometrical formula, Eq. 25, requires only one point in each span plus one more point in the span where the deformation occurs. This simple formula, therefore, makes the influence line a most handy tool for all designs involving the laws of statics. For beams with haunches, the results obtained for prismatic members are modified by values selected from precalculated tables.

A demonstration of the reduction of work made possible by a true geometrical analysis of influence lines is presented in Table 11, for comparison with Fig. 11. Let $k = i/L$ equal the stiffness; S and S', the relative stiffness (resistance to rotation) at the left and right ends of the various spans; and $S'_n + S_n + 1 = K_j$, the joint stiffness at a support. Also, m is the ratio of end rotations. The computation of m and S begins at point D of span DE and progresses toward the left; and the computation of m' and S' begins at point B in span BA, progressing to the right. Table 11(a) includes a computation for support D, which is not included in Fig. 11. By moment-distribution methods this would require the same labor as that for support B.

It is thus shown that, by geometrical analysis, the stiffness percentages at both sides of supports can be obtained directly, and accurately, with less than one fifth of the work required by the distribution method, and with about one tenth the chance of error.

In Table 11(b) the same continuous beam has been analyzed, but with support A fixed. Values of m and S are the same as for the simply supported condition (Table 11(a)). Incidentally, these results show that, by fixing point A, the stiffness distribution is changed considerably at point B but scarcely at points C and D.

The foregoing shows that geometrical analysis is not made more complex by the fixing of exterior supports. Furthermore, it shows that the concepts of statical indeterminacy and redundancy are man-made inventions which are uncalled for in design procedure and therefore obsolete. All in all, the writer agrees with the author that, in future design of rigid, or continuous, structures, the influence line will be a most valuable tool, but only when it is applied in the fundamental geometrical sense, as presented herein.[18,19,20,21]

Correction for *Transactions:* In September, 1947, *Proceedings,* on page 1014, Eq. 10 should read:

$$y_x = y_A + (y_B - y_A)(k_1)(1 - k^2{}_2 + k_1\,k_2) + l\,(\theta_A\,k_1\,k^2{}_2 - \theta_B\,k^2{}_1\,k_2)$$

and on page 1015, delete the sentence preceding the heading, "Location of Maximum Load Point."

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[18] *Transactions,* ASCE, Vol. 105, 1940, p. 1019.
[19] *Ibid.,* p. 65.
[20] *Ibid.,* Vol. 107, 1942, p. 944.
[21] *Journal of the Franklin Institute,* December, 1941, p. 553.

TABLE 11.—GEOMETRICAL ANALYSIS OF A CONTINUOUS BEAM

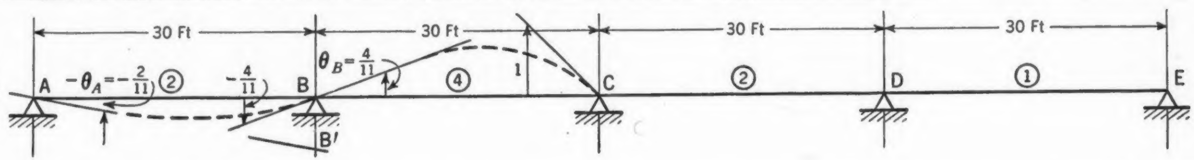
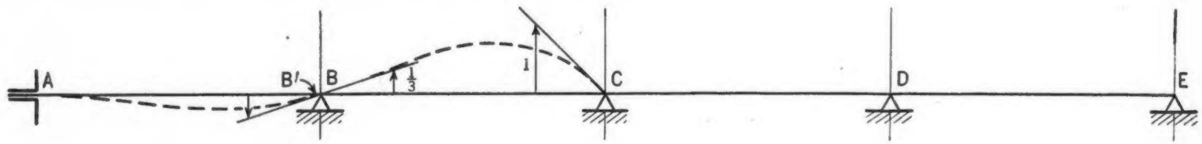
Quantity	Supports: A		B		C		D	E
(a) BEAM ABCDE, SIMPLY SUPPORTED AS IN FIG. 11								
$K = \frac{I}{l}$								
$m = \frac{\theta'}{\theta} = 0.5 \frac{k}{k + S'}$			$\frac{11}{31}$		$\frac{4}{11}$	$\leftarrow$	$\frac{1}{2}$	
$m' = \frac{\theta}{\theta'} = 0.5 \frac{k}{k + S}$		$\frac{1}{2}$	$\rightarrow$	$\frac{0.5 \times 4}{4 + \frac{3}{2}} = \frac{4}{11}$		$\frac{0.5 \times 2}{2 + \frac{36}{11}} = \frac{11}{58}$		
$S = k\,(1 - 0.5\,m)$			$\frac{102}{31}$		$\frac{18}{11}$	$\leftarrow$	$\frac{3}{4}$	
$S' = k\,(1 - 0.5\,m')$		$2\left(1 - \frac{0.5}{2}\right) = \frac{3}{2}$	$\rightarrow$	$4\left(1 - \frac{0.5 \times 4}{11}\right) = \frac{36}{11}$		$2\left(1 - \frac{0.5 \times 11}{58}\right) = \frac{105}{58}$		
$S'_n + S_{n+1} = K$		$\frac{297}{62}$		$\frac{54}{11}$		$\frac{594}{232}$		
$\frac{S'_n}{K}$; $\frac{S_{n+1}}{K}$		$\frac{3 \times 31}{297} = 0.3131$	$\frac{102 \times 2}{297} = 0.6869$	$\frac{36}{54} = 0.6667$	$\frac{18}{54} = 0.3333$	$\frac{420}{594} = 0.7071$	$\frac{174}{594} = 0.2929$	

TABLE 11.—(*Continued*)

(*b*) Beam ABCDE, Fixed at Support A (Values of m and S Are the Same as in Table 11(*a*))



Quantity	Supports: A–B	B–C	C–D	D–E
$m'\frac{\theta}{\theta'} = 0.5\frac{k}{k+S}$	$0 \longrightarrow$	$\frac{0.5 \times 4}{4+2} = \frac{1}{3}$	$\frac{0.5 \times 2}{2+\frac{10}{3}} = \frac{3}{16}$	
$S'k\,(1 - 0.5\,m')$	$2\,(1-0) = 2 \longrightarrow$	$4\left(1 - \frac{0.5}{3}\right) = \frac{10}{3}$	$2\left(1 - \frac{0.5 \times 3}{16}\right) = \frac{29}{16}$	
$S'_n + S_{n+1} = K$	$2 + \frac{102}{31} = \frac{164}{31}$	$\frac{10}{3} + \frac{18}{11} = \frac{164}{33}$	$\frac{29}{16} + \frac{3}{4} = \frac{41}{16}$	
$\frac{S'_n}{K}\,;\,\frac{S_{n+1}}{K}$	$\frac{62}{164} = 0.378$	$\frac{102}{164} = 0.622$; $\frac{110}{164} = 0.6708$	$\frac{54}{164} = 0.3292$; $\frac{29}{41} = 0.7073$	$\frac{12}{41} = 0.2927$

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DISCUSSIONS

DRAINAGE OF AIRPORT SURFACES—SOME BASIC DESIGN CONSIDERATIONS

Discussion

By H. M. Williams, W. I. Hicks, E. J. A. Gain, E. F. Brater, and Carl F. Izzard

H. M. Williams,[11] Assoc. M. ASCE.—With the increasing tendency for airport planning, design, and construction to become major fields of endeavor for the private engineer, there is a definite need for such papers as Mr. Jens' excellent contribution on airport drainage, which crystallizes recent technical developments. Certain phases of the paper, however, do excite further discussion.

The assertion that airfield drainage areas have "accurately determinable characteristics" (see "General Discussion of Method") should be viewed with discretion. Such features may be accurately determined on the drafting table, but when translated to field construction and field maintenance conditions they frequently suffer major divergencies from the design plans. In recent field drainage investigations by the Corps of Engineers, United States Army, inability to define drainage boundaries permanently and accurately has been encountered—even where low dikes or artificial drainage divides were constructed with a degree of engineering control not usually possible in normal construction operations. Inability to maintain constant drainage areas usually resulted from vehicular or airplane traffic over the turfed areas, or from the gradual consolidation and deterioration of the small drainage divides. Rutting by wheeled traffic has also been observed to cause concentration or channeling of flow with a resultant change in the runoff characteristics of the drainage area. Erosion, in some cases, resulted in increasing or decreasing the design drainage area. In one unusual condition a turfed airfield test area about 75 acres in extent frequently indicated runoff in excess of 100%, despite careful and extensive investigation and examination of automatic measuring devices, drainage divides, topography, soils, and ground-water conditions. In connection with paved surfaces, several instances have been noted where an

Note.—This paper by Stifel W. Jens was published in September, 1947, *Proceedings*.

[11] Senior Engr., U. S. Corps of Engrs., Washington, D. C.

apparent increase in drainage area resulted from wind driving surface water over low drainage divides. The foregoing data are presented, not to discredit the desirable design refinements suggested in this excellent paper, but rather to remind the reader of the wide gap that frequently exists between the design and the actual construction and functioning of field installations. It is suggested that use of reasonable safety factors, based on sound engineering judgment and experience, must still be provided, despite the niceties of design justified by recent hydrologic research.

Reference is made to the method used in several recent airport drainage papers (including the author's) for determining the outflow peak discharge by subtracting the available storage from the peak of the inflow hydrograph. The method assumes a constant head-discharge relation on the drain inlet and results in an outflow hydrograph with an extended flat peak (see Fig. 8). This assumption is an acknowledged approximation that has hitherto been considered not to introduce serious error. However, current stüdies in the Airfields Branch Office of the Chief of Engineers on design methods for combination drains have indicated that, when a varying head-discharge relation more nearly reflecting field conditions is used, a considerable increase in the peak outflow may be expected over that obtained by the author's method.

In the case of the combination drain it was noted that the peak discharge obtained by routing with the variable head-discharge relation was approximately 1.5 times the average discharge obtained by the author's method for conditions where the ratio of volume of storage to supply was greater than 30%. Between storage supply ratios of 30% and zero, the ratio of the routed peak to the average peak apparently decreased from 1.5 to unity as a straight-line relation. In the case of a typical field inlet where the flow is collected at a point rather then along a length of combination drains, it is believed that the above ratio of 1.5 reduces to approximately 1.3. This relation conforms, in general, with the results of other routing studies.[12]

The writer is not fully convinced of the necessity of using storm patterns in the design of airfield drainage systems. Inability to predetermine infiltration values accurately, the "leveling effect" on the hydrograph of a relatively small amount of storage, and the major divergencies between the storm pattern for a given occurrence and the average pattern, all contribute to this conviction. However, when justification for using storm patterns is present, the amount of work required in developing the pattern may discourage its use, particularly in the case of small projects with limited engineering funds. In such cases it appears possible to approximate the storm pattern with a minimum amount of effort by an analysis of the rainfall-duration relation for the locality. This approach is suggested by the similarity between the procedure used by the author to secure intensities for storm patterns and the normal procedure for determining the intensity-duration curve. Disregarding pattern arrangement and considering only intensities, the author's method indicates the design frequency only for the selected critical period (other periods having intensities generally less than design frequency), whereas the use of the intensity-duration

[12] "Hydraulics of Miami Flood Control Project," *Technical Report*, Miami Conservancy Dist., Dayton, Ohio, Pt. VII, p. 91.

curve will give design frequency for all periods. However, the differences resulting from use of the intensity-duration curve method do not appear to be serious, as indicated by the four rainfall pattern comparisons shown in Fig. 14, developed both by the author's method and by the intensity-duration curve.

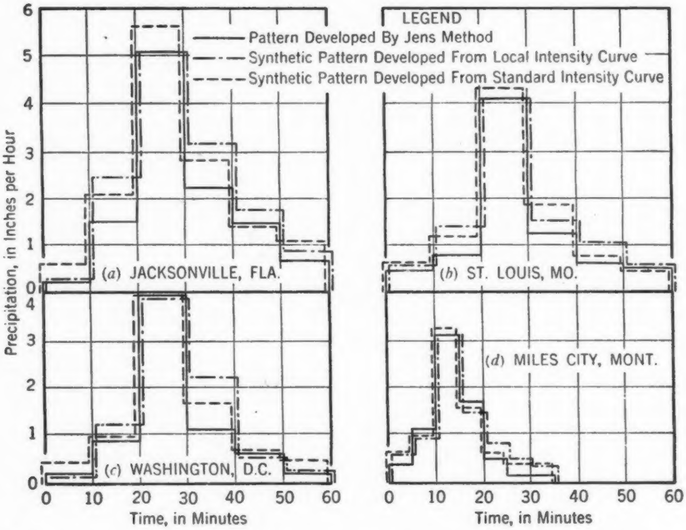

FIG. 14.—COMPARISON OF RAINFALL PATTERNS DEVELOPED BY VARIOUS METHODS FOR SELECTED LOCALITIES

In Fig. 14, it should be noted that the local intensity curves are based on 10-year records except for St. Louis Mo., where Fig. 2 was used. A typical computation for the synthesized pattern is indicated in Table 10.

TABLE 10.—TYPICAL COMPUTATIONS FOR STORM PATTERN INTENSITIES, BASED ON INTENSITY-DURATION CURVE DEVELOPED FROM 10-YEAR RECORD AT MILES CITY, MONT.

Time (min.) (1)	Intensity[a] (in. per hr) (2)	Total rainfall (in.) (3)	Rainfall increment (in.) (4)	Intensity for 5-min period (in. per hr) (5)
5	3.12	0.26	0.26	3.12
10	2.30	0.38	0.12	1.44
15	1.85	0.46	0.08	0.96
20	1.60	0.53	0.07	0.84
25	1.39	0.58	0.05	0.60
30	1.24	0.62	0.04	0.48
35	1.12	0.65	0.03	0.36
40	1.02	0.68	0.03	0.36

[a] From intensity-duration curve.

The order in which the various intensities are to be assembled into a complete pattern can be obtained by a brief study of the relative magnitudes of the component parts of a few typical storm patterns. The procedure is illustrated in Table 11 for Miles City, Mont. It is interesting to note the similarity of

the pattern arrangement obtained in Fig. 14, regardless of the widely separated geographical locations involved. Also of interest is the reasonable pattern obtained by use of the standard rainfall intensity-duration curves proposed by G. A. Hathaway,[4] M. ASCE, with the resultant elimination of the work necessary to establish the intensity-duration curve.

TABLE 11.—DETERMINATION OF TYPICAL STORM PATTERN FOR MILES CITY, MONT.

Date of storm	Storm Pattern Arrangement (5-Min Periods)							
	Preceding Periods			Maximum period	Subsequent Periods			
	3	2	1		1	2	3	4
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
6/10/25	....	....	2	1	3	....	....	....
6/15/25	....	....	3	1	2	....	....	....
8/10/25	....	....	3	1	2	....	....	....
8/28/25	....	....	3	1	2	....	....	....
8/18/26	....	....	4	1	5	3	2	6
7/8/30	....	....	3	1	2	4	....	....
9/8/30	....	....	3	1	2	4	5	....
4/21/32	3	2	4	1	6	5	....	....
6/7/32	....	....	....	1	2	3	....	....
7/19/32	6	8	7	1	2	3	4	5
8/16/32	....	3	4	1	2	5	7	6
7/10/35	....	4	3	1	2	5	6	....
6/6/28	....	....	....	1	2	....	....	....
7/28/28	....	....	3	1	2	4	5	....
Total	9	17	42	14	36	36	29	17
No. of items	2	4	12	14	14	9	6	3
Total No. of items	4.5	4.2	3.5	1	2.6	4.0	4.8	5.7
Pattern arrangement	6	5	3	1	2	4	7	8

It should be noted that the rainfall data used for this study were taken from the annual weather bureau reports for the period prior to 1936. The method of reporting excessive intensities used since 1936 makes the use of these later records exceedingly difficult for developing storm patterns by the author's method. However, it should be noted that intensity-duration curves developed from records prior to 1936 are normally low by from 5% to 10%, as pointed out by the late D. L. Yarnell,[13] M. ASCE.

W. I. HICKS,[14] ESQ.—In this exposition of the fundamental principles used in drainage design for the Lambert-Municipal Airport at St. Louis (Mo.), it should be noted that the computations are based on data for a specific location, and that the data probably will be altered elsewhere. The method is based on utilization of a depressed turfed area, lying between paved runways, for the reduction of runoff volume by soil infiltration and for the reduction of runoff intensity by ponding. The interposition of open soil between the paved area and the drainage inlet allows the maximum use of these reductive factors.

[4] "Design of Drainage Facilities," by Gail A. Hathaway in "Military Airfields: A Symposium," *Transactions*, ASCE, Vol. 110, 1945, p. 697.

[13] "Rainfall Intensity-Frequency Data," by D. L. Yarnell, *Miscellaneous Publication No. 204*, U.S.D.A., Washington, D. C., August, 1935.

[14] Civ. Engr., Storm Drain Div., Dept. of Public Works, Bureau of Eng., Los Angeles, Calif.

The paper shows conclusively that, when turfed impounding areas are available, a substantial saving in cost can be effected in drainage construction.

The method of developing the supply curve (Table 1) departs from the conventional analysis, which would involve computation of net rainfall for the significant rainstorms of record, development of a probability series for the different time durations, and determination of the predominant pattern of net rainfall intensities. The author has used a method of averaging gross and net rainfall of storms selected so as to make the average of the gross rainfall intensity identical with the two-year rainfall-duration curve in Fig. 2. The frequency of the selected values of gross rainfall intensity ranges from about ten years to less than one year (as determined by W. W. Horner, Past-President, ASCE, and F. L. Flynt,[15] Assoc. M. ASCE).

Comparison of the net rainfall patterns of the individual storms in Table 1 with the uniform rate pattern shows wide divergence with durations of individual storms varying from 5 min to 20 min within the significant 20-min duration. If Eq. 2 is altered in form for a supply duration of 20 min, a length of 185 ft, a slope of 1.75%, and a 0% imperviousness, and if it is set up for actual durations of 20 min, 15 min, 10 min, and 5 min, the results are, for the respective durations,

$$q = \sigma \tanh^2 (0.795\ \sigma^{0.5}) \qquad (7a)$$

$$q = 1.33\ \sigma \tanh^2 (0.689\ \sigma^{0.5}) \qquad (7b)$$

$$q = 2.00\ \sigma \tanh^2 (0.561\ \sigma^{0.5}) \qquad (7c)$$

and

$$q = 4.00\ \sigma \tanh^2 (0.398\ \sigma^{0.5}) \qquad (7d)$$

In Eqs. 7, the value of σ used is that for the 20-min duration of supply.

For a 20-min duration $\sigma = 1.0$ in. per hr, the values of q for actual time durations of 20 min, 15 min, 10 min, and 5 min, respectively, are 0.435, 0.480, 0.524, and 0.576 in. per hr, with added amounts explained by the fact that the supply ceases before equilibrium flow has been established; and the excess detention above equilibrium detention for given values of q causes the intensity of runoff to increase for a short time after the supply ceases. This increase is greater for small durations of supply than for large ones. The author apparently has used the method of G. A. Hathaway,[4] M. ASCE, in deriving the values in Table 2 and Fig. 10, with possibly some alteration because of infiltration after cessation of supply. For a 20-min duration $\sigma = 2.0$ in. per hr; similar values of q (before the addition in intensity because of surplus detention at the end of supply) are 1.30, 1.50, 1.76, and 2.14 in. per hr.

These analyses show a fairly wide divergence in values of q derived from a given volume of net rainfall when its duration varies from 20 min to 5 min, and suggest the alternative of basing the conception of duration on net rainfall rather than on gross rainfall. The effect of the change in analysis would be most significant in cases of summing hydrographs where no deliberate ponding is possible.

[15] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, ASCE, Vol. 101, 1936, p. 173, Fig. 23.

The infiltration curves in Fig. 4 are reasonably safe for all soils in which artificial compaction does not occur; it might be assumed that an open sandy soil would dispose of all rainfall by infiltration, with some temporary ponding for heavy storms, if it were possible to prevent clogging of the sand by the washing in of fine detritus and oil. The shape of the curve (rate of infiltration decreasing with time), the small value of antecedent precipitation for the heavy thunderstorms prevalent in St. Louis, and the frequent occurrence of the "advanced" type of storm make the tendency toward uniform supply rates greater in St. Louis than in Los Angeles (Calif.). There the heavy antecedent precipitation and the "delayed" type of storm reduce the infiltration rate to practical uniformity, particularly for the high intensities used in design. However, Table 1 indicates no universal trend toward uniformity.

In studying the derivation of the triangular hydrograph in Figs. 7 and 8, the writer derived an expression for the time required to secure practical equilibrium (t_e) in terms of average equilibrium detention (δ_a) and supply intensity (σ). When the parenthetical expressions in Eqs. 2 and 3 reach a value of 2.45, the square of the hyperbolic tangent is approximately 0.97, and practical equilibrium has been attained

$$t_e = \frac{2.45 \times 60\, l^{0.5}}{\sqrt{3{,}520\, K\, S^{0.25}\, \sigma^{0.5}}} = \frac{2.475\, l^{0.5}}{K\, S^{0.25}\, \sigma^{0.5}} \qquad (8)$$

Using the relation, $\delta_a = \delta/K$, and a value of 2.45 for the parenthetical expression, Eqs. 3 and 8 are combined to produce

$$t_e = \frac{150\, \delta_a}{\sigma} \qquad (9)$$

The writer has conducted a series of tests of overland flow on a number of surfaces, including clipped sod.[16] An examination of the data discloses, for clipped sod, the relation,

$$t_e = \frac{96\, \delta_a}{\sigma} \qquad (10)$$

A comparison of the triangular intensity hydrograph with the one derived by Eq. 2 shows that, when the time duration of supply is less than 0.6 of the equilibrium time (as defined by Eq. 9), the intensities on the ascending leg of the triangular hydrograph exceed those of the hydrograph in Eq. 2, and the triangular hydrograph is overly safe. When the duration of supply is about 0.6 of the equilibrium time, the straight leg of the triangular hydrograph averages with the curved line from Eq. 2; and when the duration of supply equals and exceeds the time of equilibrium, the triangular hydrograph becomes increasingly deficient on the ascending leg.

For examining the effect of this divergence on the outflow curve under ponding, hydrographs were constructed for an area 50% impervious with a turfed length of 185 ft, a slope of 1.75%, and an approximate ten-year uniform supply curve for 60-min duration (3.86 in. per hr on the turfed area) using: (*a*) Eq. 2, (*b*) the triangular hydrograph derived from Eq. 2, and (*c*) Eq. 10 and the value of δ_a derived from Eq. 3.

[16] Discussion by W. I. Hicks, of "Surface Runoff Determination from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, ASCE, Vol. 107, 1942, pp. 1099, et seq.

These hydrographs are delineated in Fig. 15; in the top row are the intensity hydrographs and in the bottom row, the mass hydrographs. The intensity hydrograph in Fig. 15(*c*) is derived from the mass hydrograph, the reason being that the experimental runoff data for this case were measured volumetrically, not as intensities. The close agreement of the mass runoff curves in Figs. 15(*a*) and 15(*c*) furnishes experimental confirmation for Eq. 2, in so far as storage problems are concerned.

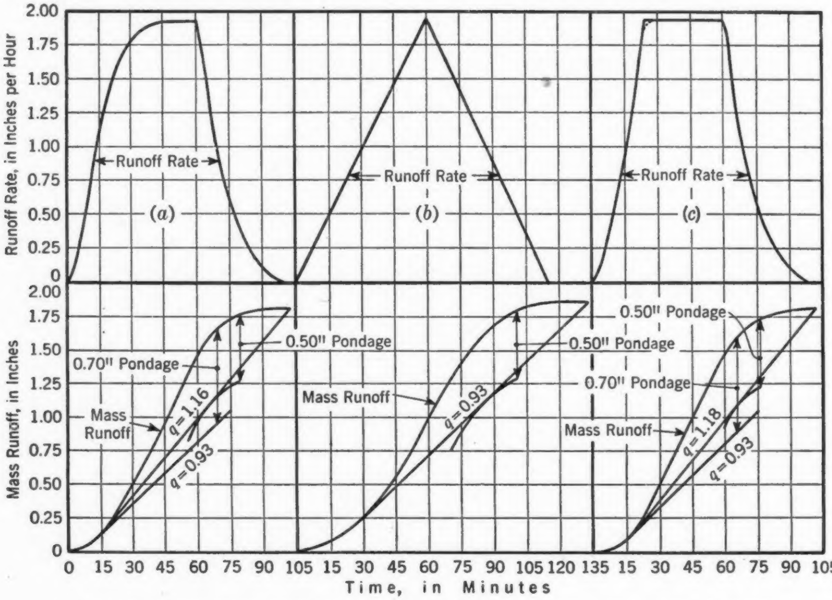

FIG. 15.—COMPARISON OF HYDROGRAPHS ON 50% IMPERVIOUS AREA WITH TURFED LENGTH OF 185 FT AND SLOPE OF 1.75%

An allowable pondage of 0.50 in. was assumed and measured downward from the mass hydrographs to form short arcs. The outflow intensity is measured as the slope of a straight line tangent to the inflow hydrograph on the accretion side and also to the short arc. The outflow intensity for the triangular hydrograph in Fig. 15(*b*) is 0.93 in. per hr, approximately 80% of the average intensity for the hydrographs in Figs. 15(*a*) and 15(*c*). The storage volume required to make the outflow intensities in Figs. 15(*a*) and 15(*c*) equal to the intensity of the hydrograph in Fig. 15(*b*) is approximately 0.70 in., or a 40% increase; the increase in depth of pondage is 12%.

The general conclusion from the foregoing analyses is that the triangular hydrograph is overly safe for durations of supply materially less than the time of equilibrium, as defined by Eq. 9, and that it is deficient in safety as the duration of supply approaches and exceeds the time of equilibrium. In the latter case, and when pondage can be developed, the deficiency can be compensated for by a temporary excess of storage.

Whereas most of the paper is devoted to the use of the triangular hydrograph in problems involving pondage, the author suggests its use (in cases where no

deliberate ponding is possible) by using basic design data and offsetting the hydrographs by the times of flow from one inlet to another. Without ponding, it is probable that a storm of ten-year intensity (or greater) would be used for design. Based on a ten-year rainfall curve (from the St. Louis data[15]), an approximate supply curve for an area 50% impervious ($\sigma = 19.05\, t^{-0.38}$) was applied to such an area with 185-ft turfed length and a slope of 1.75% for supply durations of 5 min, 10 min, 15 min, 20 min, 25 min, and 30 min. The maximum intensity of runoff was found to be 2.69 cu ft per sec per acre at 21 min for a supply duration of 20 min. The time of equilibrium (for which the parenthetical expression in Eq. 2 is equal to 2.45) is 30 min, making the time of the ascending leg of the hydrograph 70% of the time of equilibrium. For this ratio, the intensities of the hydrograph in Eq. 2 for some time prior to the peak are greater than those for the triangular hydrograph. Since, in summation, the process of offsetting hydrographs for time of flow in the drain usually requires adding the peak of the local hydrograph to the ascending leg of the drain hydrograph, use of the triangular hydrograph can lead to an unsafe cumulative error. Using a sparse turf cover, which will diminish the time of equilibrium, would accentuate such an error.

In Figs. 9 and 12 the summations of entrance head ($1.5\, h_v$) and friction head (h_f) in the tabulations are in excess of the difference between the surface elevations of the upper pond and of the pool into which the pipe system empties. This is caused by the measurement of the entrance head plus the friction head for a given length of pipe from the surface of the upstream pond to a point below the surface of the downstream pond. In Fig. 9 the entrance head is 0.34 ft and the friction head is 2.16 ft; these are measured downward from the upper pond at El. 128.34 to El. 125.84 in the downstream pond. If the full velocity head of 0.24 ft were recovered, the lower pond could not rise above El. 126.08, whereas El. 126.30 is used for the next reach of pipe. The elevations on the two profiles have been computed precisely and it would be helpful if the author were to elaborate on his computation of the hydraulic grade lines.

To summarize, the writer suggests the use of an alternative method of determining the frequency and intensity pattern of the net rainfall, and the exclusion of the triangular hydrograph in cases where ponding cannot be used. The flexibility of the ponding method of design allows the use of the triangular hydrograph with its comparative ease of computation. The fundamental conception of the full use of infiltration and pondage in reducing the cost of drainage structures cannot be questioned.

E. J. A. Gain,[17] M. ASCE.—The methods given by the author were used by him to design the drainage system for the expansion of the Lambert-Municipal Airport at St. Louis, Mo., and were presented for approval in the unpublished "Report on Basis of Design for the Field and Off-Field Drainage Systems." Since the proposed methods were relatively new and had no precedent in the sewer design practice of the City of St. Louis, they were examined critically and reviewed with respect to their fundamental equations, assumptions, basic data, and their proposed uses.

[17] Engr. in Chg. of Sewer Design, Board of Public Service, Dept. of the President, St. Louis, Mo.

In his recommendations to the chief engineer of sewers and paving (Guy Brown, M. ASCE) the writer presented the following summary to indicate the author's basis of design and its departure from current practice:

1. It is proposed to design drainage facilities for the runoff produced at the following rainfall rates—

Terrain	Frequency
Field areas	2 years
Terminal area	10 years
Off-field area	10 years

2. The rainfall available for storm runoff is termed the "supply" and is equal to rainfall minus losses due to soil infiltration, depression storage, and surface retention on plant cover. Variation of rainfall rates within the period of assumed uniform design rate and the effect of preceding rainfall on infiltration capacities are recognized factors affecting the supply.

3. Supply develops into surface runoff flowing to a collecting pond or inlet, or to the margin of a collecting stream or channel according to equations of surface flow developed by the late Robert E. Horton, M. ASCE. The effects of (*a*) length of overland flow, (*b*) slope, and (*c*) retention are recognized factors in the equation.

4. Variations in the supply rates are reflected in variable runoff rates which are reduced in intensity during the period of overland flow by the retarding effects of overland surfaces and by the storage capacity of the flowing sheet of runoff.

5. These variations in rates are further modified and their fluctuations reduced during the period of stream or channel flow by the storage capacity of the channel so that, by the time that runoff reaches the entrance of the inlet or pipe system, the large fluctuations in rainfall rates have been reduced and leveled to rates which are substantially uniform for considerable lengths of time.

6. The proposed basis of design as outlined in paragraphs from 1 to 5 is based on the work of Mr. Horton.[6,18,19] Experimental work by C. F. Izzard, Assoc. M. ASCE, of the United States Public Roads Administration, suggests that Mr. Horton's equations do not hold strictly true for all conditions of flow and that further research is necessary. Nevertheless, the Horton equations are the best that are available and the method is much better than any of the former methods in use. It differs essentially from older methods in current use in that the transformation of rainfall of a given rate to design flow of a determined rate follows the same course as that which occurs in nature, with recognition being given to all the factors modifying that course (within the bounds of existing knowledge, data, and economic expediency). The older methods usually simplified the transformation by using a given percentage of

[6] "The Interpretation and Application of Runoff Plat Experiments," by R. E. Horton, *Proceedings*, Soils Science Soc. of America, Vol. 3, 1938, p. 340.

[18] "Analysis of the Hydrograph," by Robert E. Horton, *Publication No. 101*, Soils Science Soc. of America, 1935, p. 73.

[19] "Analysis of Runoff Plat Experiments with Varying Infiltration Capacity," by R. E. Horton, *Transactions*, Am. Geophysical Union, Pt. IV, 1939, pp. 693–711.

the rainfall rate to give the design unit flow. Slope, length of flow, shape of drainage area, and kind of surface cover were acknowledged only in a general way to cover only a few general cases.

7. The proposed design of field drainage embodies the use of storage ponds in interrunway areas, thereby enabling a reduction in size of required pipe lines for design flow and also providing considerable reserve capacity for the retention of excessive runoff without greatly exceeding design limits. By their use, lines designed for a runoff from a two-year storm can accommodate a ten-year storm with only a slight rise in pond elevation.

(Steps 8 to 10 are omitted from this discussion.)

11. The basis of design proposed in this paper follows more closely the conditions that actually occur in nature than any system used before because recognition is given to all the factors pertaining to them which are known or recognized. It is in keeping with the most recent advances in hydrology and their application to design, reducing the range of guesswork and allowances for the unknown or uncertain factors entering into a design; and it makes possible a design that is rationally expected to function more closely as designed. It thus makes possible a design that will be more economical for a given set of design assumptions than any other so far proposed.

Viewed as a whole, the author's methods are an excellent example of the application of two factors in drainage which, until about 1938, received little attention—namely:

a. Determination of realistic values of runoff from a storm of a given frequency by taking into account the infiltration, surface slope, kind of surface, depression storage, and surface storage of the flowing sheet; and

b. Allowance of storage whether in sheet flow, ponds, scuppers, channels, or sewers in determining the design capacities of pipe lines or storm channels. His work is a distinct contribution in furthering the use of more refined methods of drainage design.

In application, the methods require a high degree of completion of designed surface elevations. To know the size of subdrainage areas is not sufficient. This must be supplemented by additional information concerning the site, ground slopes, ponding areas, type of surface, and allowable distance and elevations. Changes in the latter can affect the design even if there is no change in the size of the subarea. The necessary computations are considerably increased in complexity, in detail, and in amount. This is a condition that usually characterizes refined methods in design, which are justified by the following results—(*a*) closer conformity of predicted to actual functioning; (*b*) greater economy in initial cost and in operation; and (*c*) the intangible satisfaction of the designer with the results of his work. Although, in many cases encountered in municipal engineering, the increased work involved may be a deterrent to its wider application (and so may help to retain the wide use of the more easily applied but less accurate so-called rational method), the increased accuracy of the newer method can be combined with the greater ease of application of the rational method by using the former to determine a convenient and full range of P-values in the well-known formula, $Q = A\,P\,I$, for the many

typical conditions common in the larger municipalities. In this equation P is the ratio of runoff to rainfall for given conditions.

The use of designed ponds has special application to airport drainage and relatively little to municipal practice, except possibly for large park areas under limited conditions. The saving in cost of drainage by their use can be determined by noting in Table 7 the considerable difference between inflow rates and outflow rates. This indicates that, with ponding, pipe sizes can be reduced to accommodate storm flows that average from one fourth to one third of the values accumulated without ponding. Although individual values vary considerably from the average for the entire line, they indicate clearly the importance of using ponding whenever it is feasible.

There is a great need for necessary research to supplement the relatively small amount of available data required for a satisfactory application of the newer methods, and to confirm the data that have already been obtained. Available records, particularly of rainfall, need to be retabulated to put them in the detailed form that is needed for newer uses. Fortunately, the work that is necessary is not wasted since it is also applicable to other related problems. Data concerning local rainfall patterns, kinds of soil, and infiltration values for different soils and different cover are required; but, except for rainfall data, such information is not generally available, even for the larger centers of population where the need is probably greatest. The information that is available leaves much to be desired in necessary data.

E. F. BRATER,[20] Assoc. M. ASCE.—This welcome addition to the literature of surface runoff gives in detail a practical technique for selecting a design rainfall pattern and infiltration capacity curves, as well as methods for determining the benefits of pondage in reducing peak discharge—all of which should prove useful to the designing engineer. Mr. Jens suggests that the form of the hydrograph may be determined by Eq. 2, which is a formula developed by the late R. E. Horton,[6,21] M. ASCE. The writer recognizes that the validity of this formula is not of major consequence in the over-all picture of Mr. Jens' paper. However, there are such definite limitations inherent in this expression that a discussion of them seems to be warranted. The writer wishes to outline briefly Mr. Horton's derivation of this formula and then mention what seem to be its more serious weaknesses. The notation of the original derivation will be changed wherever necessary to agree with that of the author.

The storage equation is written in differential form as

$$\sigma\, dt - q\, dt = d\delta_{av} \tag{11}$$

It is then assumed that the relationship between discharge and depth of detention at the lower margin is

$$q = K_s\, \delta^m \tag{12}$$

and that

$$\delta = K\, \delta_{avg} \tag{13}$$

[20] Associate Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[21] "Hydrologic Interrelations of Water and Soils," by R. E. Horton, *Proceedings*, Soils Science Soc. of America, Vol. 1, 1937, p. 401.

in which K_s, m, and K are constants for any given runoff plot. The value of q from Eq. 12 and the value of δ from Eq. 13 may now be substituted in Eq. 11, which gives

$$dt = \frac{1}{K}\frac{d\delta}{(\sigma - K\,\delta^m)} \qquad (14)$$

If σ is held constant and m is taken as 2, Eq. 14 may be integrated to obtain

$$t = \frac{1}{K\,K_s\,2\sqrt{\frac{\sigma}{K_s}}}\log\left[\frac{\sqrt{\frac{\sigma}{K_s}} + \delta}{\sqrt{\frac{\sigma}{K_s}} - \delta}\right] + C \qquad (15)$$

The value of C is found to be zero by assuming that $\delta = 0$ when $t = 0$. Eq. 15 may then be solved for δ, giving

$$\delta = \sqrt{\frac{\sigma}{K_s}}\frac{(e^B - 1)}{(e^B + 1)} \qquad (16)$$

in which, for simplification of typography,

$$B = 2\,K\sqrt{K_s\,\sigma} \qquad (17)$$

If Eq. 16 is introduced into Eq. 12,

$$q = \sigma\frac{(e^B - 1)^2}{(e^B + 1)^2} \qquad (18)$$

Eq. 18 may also be written in the form,

$$q = \sigma \tanh^2 2\,K\sqrt{\sigma\,K_s}\,t \qquad (19)$$

Mr. Horton then assumed that K_s could be evaluated from the Manning formula,

$$V = \frac{1.486}{n}\,r^{2/3}\,s^{1/2} \qquad (20)$$

Assuming that r may be replaced by $\delta/12$, and multiplying both sides of Eq. 20 by δ, the following expression for discharge per foot width of channel is obtained:

$$q' = \frac{1.486}{n}\left(\frac{\delta}{12}\right)^{5/3} s^{1/2} \qquad (21)$$

Converting from cubic feet per second to area-inches per hour and regrouping terms, Eq. 21 becomes

$$q = \frac{1{,}020}{n\,l}\,s^{1/2}\,\delta^{5/3} \qquad (22)$$

in which l is the length of the plot in feet. By comparing Eqs. 12 and 22, it will be seen that

$$K_s = \frac{1{,}020\,s^{1/2}}{n\,l} \qquad (23)$$

If this value of K_s is substituted in Eq. 19,

$$q = \sigma \tanh^2 \left(\frac{2\, K\, \sigma^{1/2} \sqrt{1{,}020}\, s^{1/4}\, t}{n^{1/2}\, l^{1/2}} \right) \quad \text{(24)}$$

Eq. 24 is equivalent to Eq. 2 if $\frac{2\sqrt{1{,}020}}{n^{1/2}}$ is taken as $\frac{\sqrt{3{,}520}}{60}$.

Even the least of Mr. Horton's many contributions to hydrology often served the important function of stimulating study and discussion. The preceding derivation is surely one that causes the reader to probe more deeply. The runoff process provides a most complex hydraulic problem, and it would indeed be surprising if the answer were to be found in such a simplified mathematical treatment. In the first place, as in ordinary open channel problems, there are the slope, roughness, Reynolds' number, and outlet conditions to evaluate. In addition, the inflow to the plot is not only a variable with time but it varies spatially as well. Furthermore, the flow conditions deviate even farther from normal flow conditions because the raindrops, falling on the surface with nearly zero momentum in the direction of flow, cause a readjustment in the velocity distribution. A similar effect on the velocity distribution is caused on pervious areas by the draining off as infiltration of the low velocity water in contact with the ground surface. It would seem that a correct mathematical treatment would have to include most of the foregoing conditions. Because many of them were not considered in this derivation, the effect of their omission on the accuracy of the results to be expected from the use of Eq. 24 must be considered.

The first assumption made was that supply (σ) is a constant. Even if precipitation rate were assumed to be constant, σ varies with time somewhat in the manner shown by

$$\sigma = p - c_1 - c_2\, e^{-kt} \quad \text{(25)}$$

in which p is the precipitation rate, and c_1, c_2, and k are constants. It will readily be seen that the presence of this expression for σ in Eq. 14 would have required a different integration process. Furthermore, if the integration were carried out with σ replaced by a function of t, there is no reason to believe that the result would be of the same form as Eq. 19. One very apparent consequence of the assumption that σ is a constant is that Eq. 24 shows q to become zero at the instant that σ becomes zero, thus making the recession side of the hydrograph a vertical line. Consequently, it is necessary to derive the recession curve by another method.

The second assumption was that m, the exponent of δ in Eq. 12, equals 2. This value was probably chosen by Mr. Horton because it is within the range of values of m and at the same time it made Eq. 14 easy to integrate. Eq. 14 may also be readily integrated for values of m equal to 1 and 3, but the resulting equations are quite different from Eq. 19. Although it is possible for two equations of different form to represent similar paths over limited regions, this cannot be assumed to be the case unless it has been demonstrated. It follows then that Eqs. 19 and 24 may only be used with assurance for plots having m equal to 2. The question thus arises as to whether 2 is a frequently occurring value of m. In the case of steady, uniform, laminar flow, both theory and

experimental results show that m equals 3, whereas, for turbulent flow, test results culminating in the development of the Manning formula (Eq. 20) show that m is approximately 5/3. This is illustrated by the test results plotted in Fig. 16, for which the values of q and δ were obtained from studies[22] at the U. S. Waterways Experiment Station at Vicksburg, Miss. It should be noted that the points plotted in Fig. 16 follow the slope of 3 in the laminar flow region and the slope of 5/3 in the turbulent range. In the transition range, the plotted points show that m does not change gradually from 3 to 5/3, but changes rather quickly from 3 to 1 or less, and then gradually changes to 5/3. Thus, the region in which m equals 2 seems to be limited to a short range of Reynolds' numbers. Tests reported by W. O. Ree[23] and V. J. Palmer[24] for uniform steady turbulent flow through heavy grass cover show that m is approximately 5/3 for this case also.

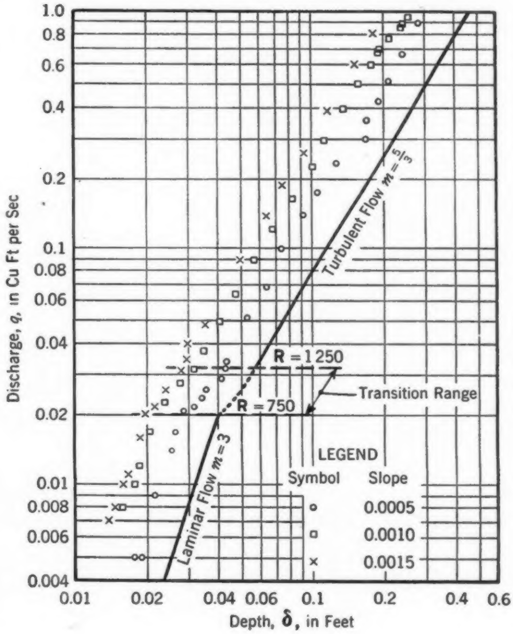

FIG. 16

One cannot predict with assurance that the relations between average depth and discharge for the unsteady conditions found on runoff plots will follow the same pattern as the uniform steady flow data plotted in Fig. 16. However, results obtained by C. F. Izzard,[10] Assoc. M. ASCE, for runoff resulting from the application of artificial rainfall on paved and turfed surfaces indicate that m equals 3 in the laminar range. Mr. Izzard's tests did not extend into the transition or turbulent ranges. The writer has encountered a number of plots[25] for which m equals 1 or less, thus indicating the presence of a transition range similar to that shown in Fig. 16.

Mr. Horton recognized that m would not usually equal 2 and suggested that Eq. 24 might still be made to give approximately correct results by a corresponding distortion of the values of K_s.[20] However, it should be noted, with reference to Fig. 16, that varying K_s simply changes the vertical position

[22] "Studies of River Bed Materials and Their Movement, With Special Reference to the Lower Mississippi River," *Paper No. 17*, U. S. Waterways Experiment Station, Vicksburg, Miss., January, 1935.

[23] "Some Experiments on Shallow Flows Over a Grassed Slope," by W. O. Ree, *Transactions*, Am. Geophysical Union, 1939, Pt. IV, p. 653.

[24] "Retardance Coefficients for Low Flow in Channels Lined with Vegetation," by Vernon J. Palmer, *ibid.*, April, 1946, p. 187.

[10] "The Surface Profile of Overland Flow," by C. F. Izzard, *ibid.*, 1944, Pt. VI, p. 959–968.

[25] "Sprinkled Plat Runoff and Infiltration Experiments on Arizona Desert Soils," by E. L. Beutner, R. R. Gaebe, and R. E. Horton, *Technical Publication No. 38*, SCS, U.S.D.A., September, 1940.

of any straight line. Therefore, a line having a slope of 2 can have but one point in common with a set of test results which falls along a slope other than 2. Consequently, a hydrograph synthesized on the basis of arbitrarily chosen values of K_s and m will arrive at the ultimate peak on a path which must be different from the correct one. If a particular rate of precipitation were to continue long enough for outflow to become equal to supply ($q = \sigma$), the error introduced is probably of little importance, but for storms of lesser duration there will be an error in the predicted peak discharge. When $q = \sigma$, no equation is needed to determine the peak discharge, because it is equal to the precipitation rate minus the infiltration capacity.

Although that part of the derivation of Eq. 24 previously discussed is based on the assumption that m equals 2, the value of K_s is determined from a formula for fully developed turbulent flow, applying only to the case where m equals 5/3. Furthermore, the expression for K_s (Eq. 24) so obtained does not apply to laminar flow, because in that case the exponent of s is 1 rather than $\frac{1}{2}$, and n does not vary with the roughness, but only with the Reynolds' number. The same limitations would apply in varying degrees to flow in the transition range.

Finally, before using Eq. 24 it is necessary to estimate K, which is the ratio of the depth at the margin to the average depth as shown by Eq. 13. Mr. Jens has used the value of 1.5 as suggested by Mr. Horton. However, K varies with the slope, outlet conditions, roughness of the surface, infiltration capacity, and rainfall intensity. For very flat surfaces the depth will increase in the upstream direction and K will be less than 1. For slopes less than critical, the profile is influenced by the presence of critical depth at the outlet, whereas for steeper slopes this is not the case. For any slope, the outlet may be drowned out, causing backwater, and in the case of slopes greater than critical, a hydraulic jump would occur. Sheet flow over irregular surfaces at high velocities is characterized by the presence of many small hydraulic jumps and standing waves. All these factors briefly mentioned heretofore combine to add to the difficulty of choosing a suitable value of K. The writer suggests that Eq. 12 could be written in terms of δ_{avg} rather than δ, thus avoiding the necessity of estimating K.

It is believed that the previous discussion shows that the derivation of Eq. 24 is based on a number of assumptions that are not entirely consistent with one another and that are likely to differ from the conditions prevailing on any particular runoff plot. Consequently, despite the fact that it was derived from the basically correct storage equation, Eq. 24 is an empirical formula. Two of the assumptions influenced the method of integration to such an extent that the form of Eq. 24 probably bears little resemblance to what might be called the correct form. The fact that K_s must be distorted to overcome arbitrary assumptions as to m introduces one of the weaknesses found in the "rational formula." Finally, it is necessary to employ some other method for deriving the recession side of the hydrograph—Eq. 24 being applicable only to the rising side. Perhaps none of these objections would be serious if the magnitude of the deviation of results obtained by the use of Eq. 24 from the correct results could be predicted for any set of conditions. Because this apparently

cannot be done, the writer prefers a method in which the effect of the various assumptions is clearly evident.

A graphical procedure which permits the derivation of both the rising and falling sides of the hydrograph, similar to that described by W. W. Horner, Past-President, ASCE, and Mr. Jens[3] in connection with gutter flow and by C. R. Hursh and the writer[26] for determining outflow from channel storage on small watersheds, might be used. In this method, the storage equation, together with a relation between discharge and average depth of detention—

$$q = K_s\, \delta^m{}_{\text{avg}} \qquad (26)$$

—are solved for successive time intervals. The constants in Eq. 26 must be determined from tests on terrain comparable to that upon which they are to be used, and are usually determined from the recession side of a hydrograph. Although the values of K_s and m are usually somewhat different for the rising and falling sides of a hydrograph, the magnitude of the error introduced in this manner can be tested by using the relation derived from the recession curve to synthesize the rising side of a known hydrograph. Many such checks made by the writer indicate that this effect is small.

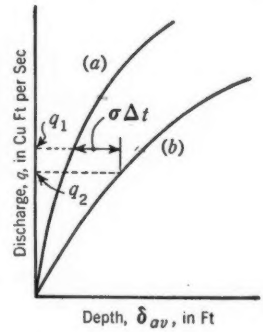

FIG. 17

The storage equation is written

$$\delta_1 + \sigma\,\Delta t - \frac{(q_1 + q_2)}{2}\,\Delta t = \delta_2 \qquad (27)$$

in which the subscripts 1 and 2 denote values occurring at the beginning and end, respectively, of a time interval Δt, and σ is the average supply during an interval. Eq. 27 may be rearranged as follows:

$$\left(\delta_1 - \frac{q_1\,\Delta t}{2}\right) + \sigma\,\Delta t = \left(\delta_2 + \frac{q_2\,\Delta t}{2}\right) \qquad (28)$$

Values of $\left(\delta - \frac{q\,\Delta t}{2}\right)$ and $\left(\delta + \frac{q\,\Delta t}{2}\right)$ may then be plotted against corresponding values of q, as illustrated by curves (a) and (b) of Fig. 17. Knowing the discharge at the beginning of any interval (q_1), the value of $\sigma\,\Delta t$ may be added to the corresponding abscissa of curve (a) and the value of q_2 read from curve (b) in the manner illustrated. The process is then repeated for successive intervals until the entire hydrograph is derived.

CARL F. IZZARD,[27] ASSOC. M. ASCE.—The main purpose of this discussion is to present pertinent data on the hydraulics of overland flow as related to airport drainage. As noted by the author, the United States Public Roads

[3] "Surface Runoff Determination from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, ASCE, Vol. 107, 1942, p. 1039.

[26] "Separating Storm-Hydrographs from Small Drainage-Areas into Surface- and Subsurface-Flow," by C. R. Hursh and E. F. Brater, *Transactions*, Am. Geophysical Union, 1941, Pt. III, p. 863.

[27] Senior Highway Engr., Public Roads Administration, Washington, D. C.

Administration conducted, with the cooperation of the United States Soil Conservation Service on the experimental phase, an investigation of overland flow on paved and turf surfaces. A report summarizing these studies[28] indicates that the rate of runoff under a uniform rate of supply can be determined for any surface by use of a single dimensionless hydrograph that takes the place of the Horton formula (Eq. 2). The latter was developed from very limited experimental data on short plots, whereas the dimensionless hydrograph integrates

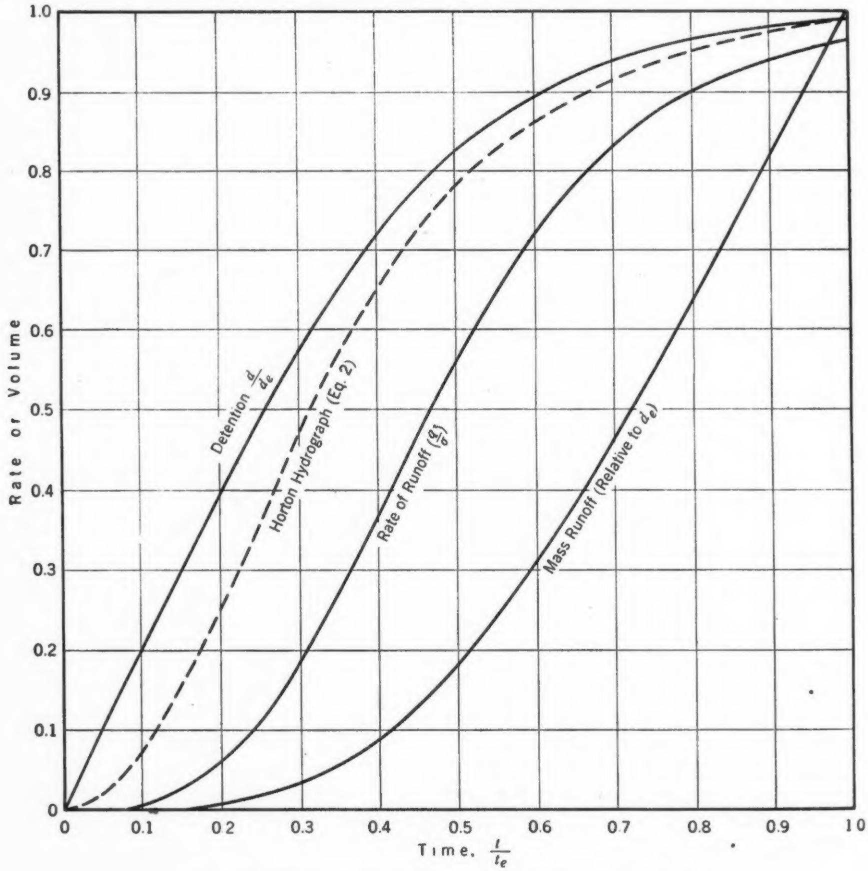

FIG. 18.—DIMENSIONLESS HYDROGRAPH OF OVERLAND FLOW

the results of several hundred tests on a variety of lengths, slopes, and surfaces with two different rainfall intensities. The principal difference between the two curves, as shown in Fig. 18, is that the Horton curve starts rising sooner after the beginning of supply. Aside from the fact that the dimensionless hydrograph affords a more accurate representation of overland runoff, it is also easier to use because a single curve takes the place of the families of curves

[28] "Hydraulics of Runoff from Developed Surfaces," by Carl F. Izzard, *Proceedings*, Highway Research Board, National Research Council, Vol. 26, 1946, pp. 129–150.

needed to facilitate use of the Horton equation. Although the dimensionless hydrograph is limited to lengths of flow and intensities of supply such that $\sigma l < 500$, this limitation includes most of the conditions occurring on interrunway areas on airfields. When experiments being conducted by the Corps of Engineers are completed, it should be possible to determine detention characteristics for fully turbulent flow beyond the limits of the Public Roads Administration experiments. However, the writer, using the dimensionless hydrograph, has been able to obtain a reasonable reproduction of an actual hydrograph on a combined pavement and turf area where the average length of overland flow was about 1,000 ft with maximum supply intensities of 5 in. per hr on turf. In this case, the peak rate of runoff did not exceed 0.5 in. per hr, so the maximum rate of flow was within the limitation of the dimensionless hydrograph. For long lengths of flow on turf, the duration of the high intensity supply rates is always a small fraction of the equilibrium time.

Use of the dimensionless hydrograph is simple. Time from beginning of supply is measured as a ratio to the time of equilibrium (t/t_e) required for the runoff rate to become substantially equal to the supply rate σ. The ordinate q/σ gives the ratio of rate of runoff to supply rate at the time t/t_e. To evaluate t_e in minutes it is necessary to know detention at equilibrium, d_e. Thus,

$$t_e = \frac{120\, d_e}{\sigma} \qquad (29)$$

in which

$$d_e = \frac{K_d\, L^{1/3}\, \sigma^{1/3}}{2.92} \qquad (30)$$

and is measured in inches; the detention coefficient K_d is expressed in terms of slope S and retardance coefficient c as

$$K_d = (0.0007\, \sigma + c)\, S^{-1/3} \qquad (31)$$

Values of c are as follows: Smooth asphaltic pavement, 0.007; crushed slate roofing paper, 0.0082; concrete pavement on airfield apron, 0.012; tar and sand surface, 0.017; clipped sod surface, 0.046; and dense bluegrass turf, 0.06.[28]

The dimensionless hydrograph can also be used to compute rates of runoff resulting from storms involving varying rates of supply. The method is described in detail elsewhere[28] and depends on the principle, developed from study of experimental data, that the rate of runoff is a function of the detention at any time. Thus, when the supply rate changes, the rate of runoff immediately adjusts to the rate which would have existed at the new supply rate with the given amount of detention. For runoff on turf, especially with pondage, which is almost always involved, the writer agrees with Mr. Jens that the average supply rate can be used to determine the peak rate, so that it is not ordinarily necessary to compute hydrographs with varying supply rates.

A dimensionless curve has also been developed to compute rates of runoff during recession.[28] This curve is in terms of a function which involves the detention and rate of runoff at the beginning of the recession so that the total area under the recession curve will equal the amount of detention. Inasmuch as the present discussion is concerned with triangular approximations of hydrographs, there is no necessity of using the dimensionless recession curve.

The Public Roads Administration experiments included a few tests on turf plots with various percentages of impervious area draining across the turf. The typical hydrograph shows a sharp increase in rate of runoff as soon as runoff from the impervious area reaches the lower margin of the turf.[28] A fair approximation of the rising hydrograph can be obtained by using the dimensionless hydrograph if the amount of detention in Eq. 29 is reduced in proportion to the reduction in detention resulting from having pavement instead of turf on the upper part of the area. A simple equation to determine this reduction is

$$\frac{d_c}{d_t} = 1 - \left(\frac{l_p}{l}\right)^{4/3} \left(1 - \frac{K_p}{K_t}\right) \qquad (32)$$

in which d_c and d_t are equilibrium detention values for the composite area and a turf area of the same length l, respectively; l_p/l is the ratio of paved to total length; and K_p and K_t are detention coefficients for the paved and turf areas, respectively. The equation thus takes into consideration different slopes on paved and turf areas, since that factor is included in Eq. 31. The detention (for the composite area) used in Eq. 29 is the detention for a turf area of the same total length multiplied by the ratio d_c/d_t. The supply rate used is the supply rate for the turf as determined by Mr. Jens (with a proportionate increase in rainfall intensity to determine infiltration on turf area) converted to equivalent supply rate for the total length of overland flow, including the paved area.

To illustrate this procedure, the overland flow length of 185 ft shown in Fig. 7 is taken, and it is assumed that the upper 50 ft are paved, making the ratio of paved to turf area 0.27. The net supply rate on the composite area is 0.85 in. per hr for 30 min. From Eq. 31, values of K_p and K_t are 0.07 and 0.233, respectively, assuming slopes of 0.01 (on concrete pavement) and 0.0175 (on dense turf) and a rainfall intensity of about 3 in. per hr (on pavement). Detention on 185 ft of turf of uniform slope is 0.43 in. based on a supply rate of 0.85 in. per hr. This value is multiplied by 0.88 (computed by Eq. 32) to obtain 0.38 in. for the detention at equilibrium on the composite area. From Eq. 29 the equilibrium time is found to be 54 min so that the value of t/t_e at the time supply ended is $30/54 = 0.56$ and that of q/σ is 0.65 from the dimensionless hydrograph. The rate of runoff at 30 min is thus $0.65 \times 0.85 = 0.55$ in. per hr.

A slight error is involved in this method because the actual supply rate on the paved area is different from that on the turf area. The error is negligible because detention on the pavement is a very small proportion of the total detention on inter-runway areas.

Commonly, the width of paved area will be greater than the average width of turf area. To allow for the increased detention volume on pavement resulting from this situation, the ratio of areas may be substituted for the ratio of lengths used in Eq. 32.

Values of q_p computed by the dimensionless hydrograph method for the supply rates given in Table 2 give closely similar results except for short durations on 0% impervious areas, where the values were about 75% of those in

Table 2. The reason for the difference is probably the fact that the Horton equation gives higher rates of runoff where the duration of supply is substantially less than the time of equilibrium. Using Eq. 32 in estimating detention on 30% impervious areas, the values of q_p are practically the same as those in Table 2 for all durations, but tend to be a little higher than those in Table 2 for the shorter durations.

With reference to the method of computing the rate of outflow from a pond as illustrated in Fig. 8, the writer, after examining actual hydrographs of runoff from ponded areas on airfields,[29] feels that the flat topped outflow hydrograph is not a good representation and that a triangular hydrograph as shown in Fig. 19 would be more nearly correct. The latter is more consistent with actual

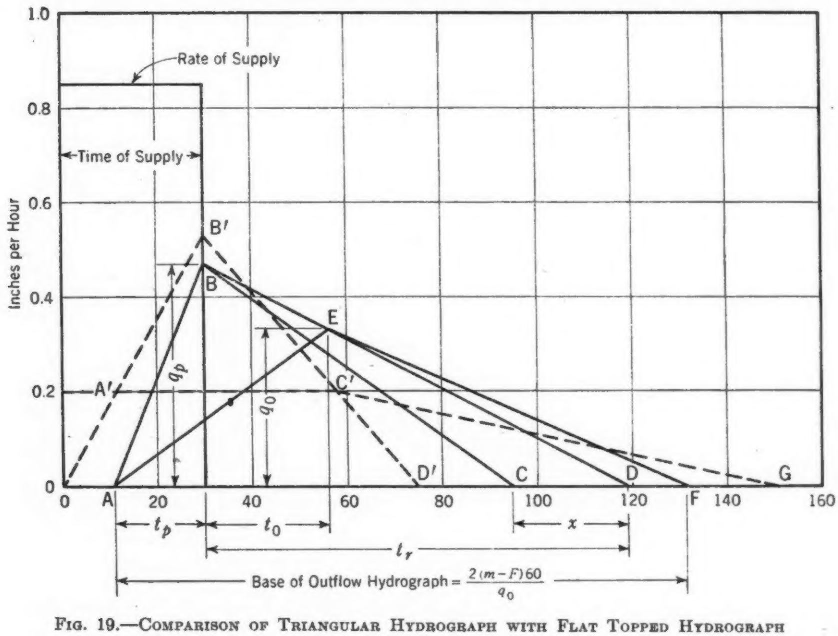

FIG. 19.—COMPARISON OF TRIANGULAR HYDROGRAPH WITH FLAT TOPPED HYDROGRAPH

hydrographs because ponding begins at the beginning of the inflow hydrograph and reaches a maximum at a time after the ocurrence of the peak rate of inflow.

The example in Fig. 7 is used to illustrate this point. The average supply rate for 30 min is 0.85 in. on 185 ft of turf on a 1.75% slope. From Eqs. 31 and 30, it is determined that $K_d = 0.233$ and $d_e = 0.431$ in., from which $t_e = 61$ min by Eq. 29. At 30 min, $t/t_e = 30/61 = 0.49$. In the dimensionless hydrograph, Fig. 18, $q/\sigma = 0.55$ for this value of t/t_e, and therefore $q_p = 0.55 \times 0.85 = 0.47$ in. per hr; also, from the detention and mass runoff curves (Fig. 18), detention $= 0.81 \times 0.431 = 0.349$ in. and mass runoff $= 0.17 \times 0.431 = 0.073$ in. The sum of detention and mass runoff is 0.422 in., which checks the amount of rainfall (0.85 in. per hr $\times \frac{1}{2}$ hr) within 0.003

[29] "Report on Drainage Verification at Military Establishments," Louisville Engr. Dist., Ohio River Div., Corps of Engineers, War Dept., Louisville, Ky., June, 1947.

in., the error being due to using only two significant figures in reading the values from the curve. Knowing q_p (0.47 in. per hr), the rising time t_p is computed as $\frac{2 \times 0.073 \times 60}{0.47} = 19$ min, which places the beginning of the runoff at 11 min after beginning of supply. The recession time t_r similarly becomes $\frac{2 \times 0.349 \times 60}{0.47} = 89$ min. The latter time can be adjusted by Mr. Jens' method for infiltration during recession. Thus, if $f = 0.26$ in. per hr, by Eq. 5, $X = \frac{89 \times 0.26}{(1.5 \times 0.47) + 0.26} = 24$ min.

The peak rate of outflow is estimated as

$$q_o = q_p \left(1 - \frac{p}{m}\right) \quad \text{.....(33)}$$

in which p is represented by the area ABE (Fig. 19) and m is the runoff in inches before deducting infiltration during recession on the assumption that the total supply is effective in producing the peak outflow. The time after end of supply at which peak outflow occurs is

$$t_o = t_r \left(\frac{p}{m}\right) \quad \text{.....(34)}$$

In the example, if $p = 0.125$ in., then $\frac{p}{m} = \frac{0.125}{0.425} = 0.29$ and $q_o = 0.47 \times 0.71 = 0.33$ in. per hr, occurring at $t_o = 89 \times 0.29 = 26$ min after the end of supply. The base of the outflow hydrograph must be such that the area equals supply minus infiltration during recession. The latter amounts to $\frac{0.47 \times 24}{2 \times 60} = 0.094$ in., making net runoff $= 0.425 - 0.094 = 0.331$ in. The base of the outflow hydrograph, therefore, is $\frac{2 \times 0.331 \times 60}{0.33} = 120$ min.

For comparison, the dotted lines show the inflow and outflow hydrographs which would result from using the same assumptions in Mr. Jens' method. Triangle AEF is believed to be a more true representation of outflow from a pond, because the peak occurs after supply has ended, which is necessarily the case, whereas the flat peak A′C′ begins before any substantial runoff has occurred. Summation of a series of hydrographs with flat tops would give an excessively high rate of runoff early in the storm. Summation of triangular outflow hydrographs would come closer to giving true peak outflow rates, if hydrographs are offset to allow for routing through conduits.

Mr. Jens does not indicate how the ponded water is discharged during recession. In Fig. 19, this has been shown as C′G with area C′D′G equal to area A′B′C′, the volume of pondage. In Fig. 13, the recession curves are not shown as straight lines, which raises a question as to what method Mr. Jens used for computing recession curves.

The writer was unable to determine how the use of the triangular outflow hydrograph would affect the final design of a system, since insufficient basic

data were given in the example. It would be instructive if the author could recompute one set of data by this principle giving the results in his closing discussion.

The basic concept of the triangular outflow hydrograph was used by the writer in an unpublished report written in November, 1941. The method was not applied then because no good procedure for estimating the peak rate of inflow had been developed.

The writer doubts that the method of developing average supply patterns as illustrated in Table 1 is logically correct. As an analogy, the arithmetic mean of the number of spots on a die is 3.5 but the probability of rolling 3.5 is zero. In other words, the arithmetic mean has no possibility of occurring. On the other hand, the probability of rolling any given number from 1 to 6 is 1 in 6.

The probability of a given amount of rainfall in a given duration is fairly well established from rainfall intensity-frequency data such as those compiled by the late D. L. Yarnell,[13] M. ASCE. Table 1 shows that there is considerable variation in the pattern of rainfall intensities. Certain patterns result in higher rates of runoff than others. Obviously, the probability of encountering one of the critical patterns is not the same as the probability of equaling or exceeding a given total amount of rain in a given period. For example, if the probability of the amount of rain is 1 in 2, and the probability of the pattern being critical is 1 in 3, then the resultant probability is 1 in 6. The other five storms would not be critical.

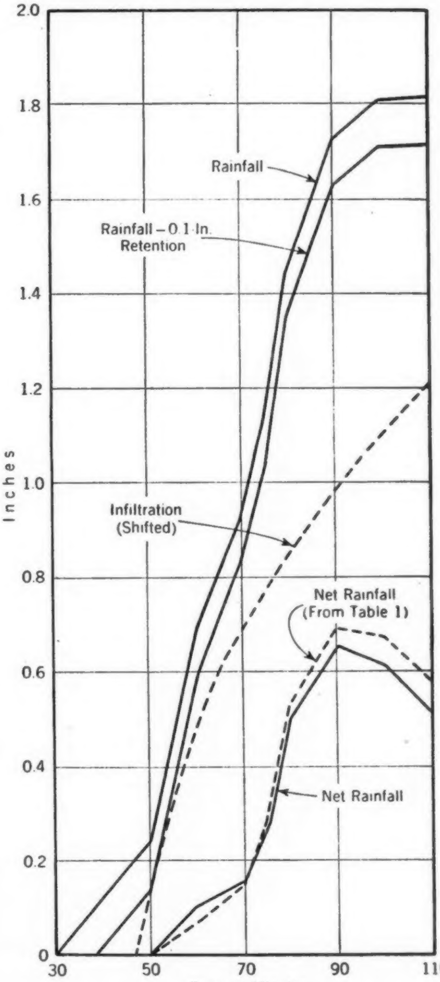

FIG. 20.—MASS CURVES FOR STORM OF SEPTEMBER 12, 1925, FROM TABLE 1

The writer is not prepared to suggest exactly how the statistical analysis should be made. That study should be undertaken by a mathematical statistician who is fully aware of the limitations of statistical methods.

The writer wishes to suggest a method of estimating the supply rates for a given rainfall intensity pattern and infiltration capacity curve which gives

results closely comparable to those in Table 1 and is easier to follow. The method requires only a mass rainfall curve, a mass infiltration curve, and an estimate of the initial retention. For example, Fig. 20 shows mass rainfall for the storm of September 12, 1925, taken from Table 1. The curve is replotted 0.1 in. lower to allow for 0.1-in. retention. Then the infiltration mass curve is moved over until it touches at one point only—namely, at 50 min. Prior to that time, rainfall intensity was lower than infiltration rate so all the rainfall was absorbed. Beyond 50 min, rainfall intensity exceeds infiltration rate and in the next 10 min the excess is 0.10 in., as shown by the net mass rainfall curve. The net rainfall rate for that period is thus 0.6 in. per hr. At 90 min, the rainfall rate drops below the infiltration rate and no additional net rainfall occurs. Whether or not infiltration capacity recovers during periods when rainfall is less than infiltration is not fully established. It might be assumed, however, that the infiltration curve could be shifted to the right until the ordinate measured to the mass rainfall less retention curve is 0.65 in. at the time when excess rainfall resumes, 0.65 in. being the maximum net mass rainfall previously attained.

In conclusion, the writer believes that the principles for estimation of peak rates of runoff from ponded inter-runway areas on airfields as suggested by Mr. Jens are fundamentally sound. These changes relate to detailed procedures and are presented so that other engineers working on similar problems may consider the alternatives and develop still better procedures.

Corrections for *Transactions:* In September, 1947, *Proceedings*, on page 986, in Table 1, line 37, Col. 10, change "0.059" to "0.59"; on page 994, in Table 2, Col. 4, for 60-min duration, change "0.774" to "0.784"; and on page 997, seventh line from bottom, change "1%" to "30%."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

INVESTIGATION OF DRAINAGE RATES AFFECTING STABILITY OF EARTH DAMS

Discussion

BY ROSS M. RIEGEL, AND T. W. LAMBE

ROSS M. RIEGEL,[16] M. ASCE.—In the design of earth dams it has been the practice of the Tenessee Valley Authority (TVA) to compute a "factor of safety" for the slope exposed to the reservoir water by the method described by the author (under the heading, "Applications"); that is, it is assumed that the pore-water pressures which would exist under conditions of drawdown would be those due to a complete lack of drainage. (It is assumed also that the pore pressure head existing at any point within the dam is the static head existing at that point, measured either to the surface of the fill, to the saturation line, or to the surface of the reservoir water—as the case may be.) Under these conditions the minimum factor of safety prescribed in the design of new structures has been about 1.25. This rather low factor has been used largely because it has been believed that the assumed criterion of no drainage was actually more severe than the probable true condition; the information necessary for a more accurate appraisal of true conditions was wanting. The author's paper remedies this deficiency to a high degree, and enables the problem of design to be approached with more confidence.

It has also been the practice of the TVA to design the downstream slope, which was usually relatively free from uncertainty as to the effects of drainage, with a factor of safety of at least 1.5. This is an index of a more proper value when the drainage conditions are well understood and can be evaluated through the methods developed by the author. The effect of more accurate drainage treatment on the computed factor is strikingly illustrated by the application to Blue Ridge Dam in northwest Georgia, cited by the author.

The determination of the slope of major earth structures has also been subject to the review of the Board of Consulting Engineers (TVA) whose experience and judgment have been material factors in the final determination

NOTE.—This paper by F. H. Kellogg was published in September, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1948, by Harry R. Cedergren.

[16] Head Civ. Engr., Design Div., TVA, Knoxville, Tenn.

of slopes. It is suggested that the factor of safety must still have a material value, even after the most accurate design methods available have been employed. Some allowance must be made for two points. The material actually placed in the fill may depart in its characteristics from those determined from the occasional samples examined in the soils laboratory. The construction process, to some degree, may also fail to develop the anticipated characteristics as to compaction, moisture content, friction angle, and cohesion, which are set up in the laboratory investigation and design assumptions.

The author is to be complimented on shedding much light upon an obscure subject. His investigations of full-size structures have been based on embankment materials which may be generally classified as clays, whereas his model experiments have employed relatively coarse sand. He points out that, from the standpoint of drainage, soils classified as silts and fine sands are probably the least satisfactory embankment materials; yet, embankments have been built from them. It would be most interesting if the application of the author's methods of experiment and analysis to existing conditions in such structures would be undertaken, and it is suggested that such an investigation would be a valuable addition to this subject.

T. W. LAMBE,[17] JUN. ASCE.—The method of computing rates of drainage and the magnitudes of total water head in the draining soil mass, based on an analogy of drainage with heat flow or consolidation, is an interesting approach to the drainage problem. In this discussion the writer will present comparative studies of the author's method, and various other theoretical methods, with the results from laboratory tests (made during a research program carried out by the writer which furnished material for a thesis to be submitted in partial fulfilment of the requirements for the degree of Doctor of Science at Massachusetts Institute of Technology in Cambridge), in which a soil was actually drained. Also presented will be a rigorous theoretical derivation of the author's method for vertical drainage, so as to show more clearly some of the assumptions and conditions involved.

Fig. 14 is a plot of drainage quantity, expressed as a percentage of both total soil volume and soil void volume of the draining mass, against time, expressed as a dimensionless time factor, T, for a laboratory test and for various theoretical solutions. The methods used to obtain the separate curves are explained first briefly and later analyzed after the theory has been presented so that the meaning of their respective assumptions can be more fully appreciated. Curve (*b*) was plotted from data taken during the bottom drainage of a 6-ft vertical tube of a natural fine, uniform, saturated sand. Theory C is a solution of Eq. 21*a* with a more reasonable assumption of initial total head distribution than that used by the author in obtaining Eq. 25*a*. Curve (*c*) was obtained by use of equations developed by Karl Terzaghi,[18] Hon. M. ASCE. Curve (*d*), for transient flow nets,[19] was plotted using steady state flow nets at the various positions of the falling line of saturation.

[17] Instr., Soil Mechanics, Civ. Eng. Dept., Mass. Inst. Tech., Cambridge, Mass.

[18] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 314, Eqs. (9) and (10).

[19] "Retaining-Wall Design for Fifteen-Mile Falls Dam," by Karl Terzaghi, *Engineering News-Record*, May 17, 1934, p. 532.

Eq. 25*a* was not plotted in Fig. 14 because the writer knew no reasonable way to compute the ultimate drainage quantity due to the initial total head assumption used to derive Eq. 25*a*. This point is explained more fully later.

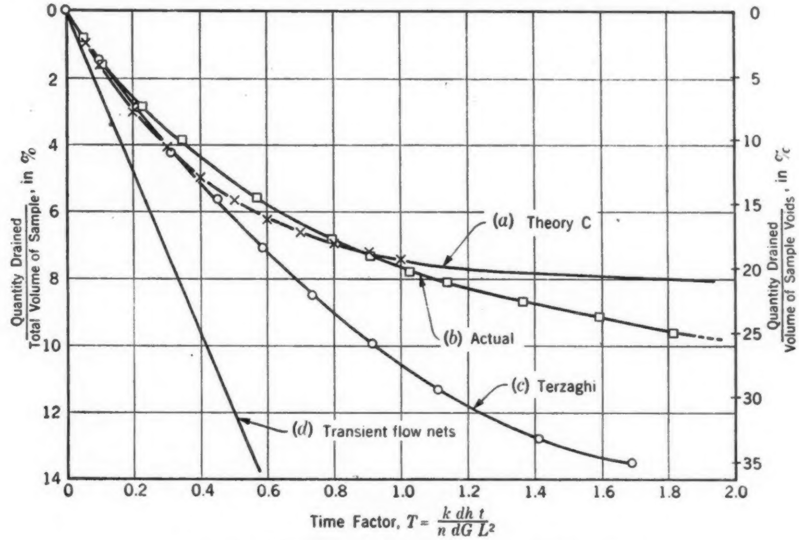

FIG. 14.—DRAINAGE QUANTITY VERSUS TIME FACTOR

However, in Fig. 15, Eq. 25*a* is compared with theory C, for both methods, by plotting T against the percentage of drainage, where the percentage of drainage is assumed equal to the percentage of the average total head which has been

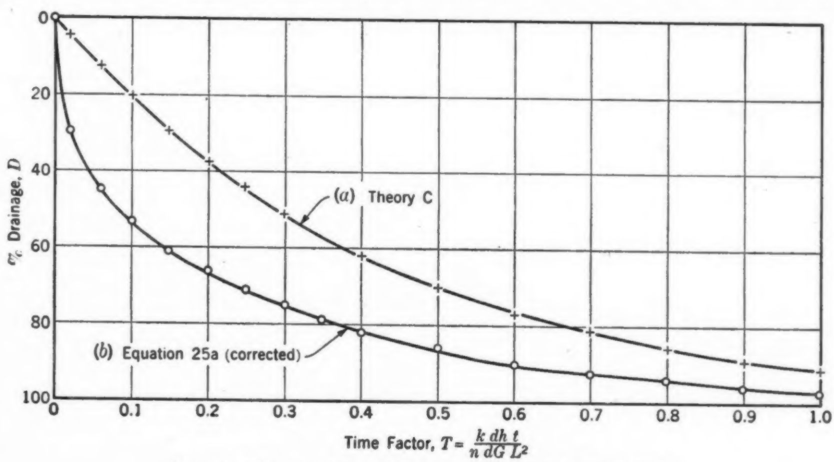

FIG. 15.—PERCENTAGE OF DRAINAGE VERSUS TIME FACTOR

dissipated. An examination of Figs. 14 and 15 shows that: (*a*) Theory C more nearly approximates the actual flow curve than does Eq. 25*a*; and (*b*) none of the theoretical methods correctly takes care of the actual time lags that exist.

Before considering the theoretical methods, a brief description of the true drainage process is given so that the validity of the various assumptions which are made to obtain solutions can be better evaluated. Fig. 16 is a plot of total heads in the sand sample at various times during drainage. (The datum for measuring elevation heads was taken at the bottom of the soil, which is also the atmospheric pressure line. The convention is followed throughout this discussion.) The times are represented by the dimensionless time factor, T,

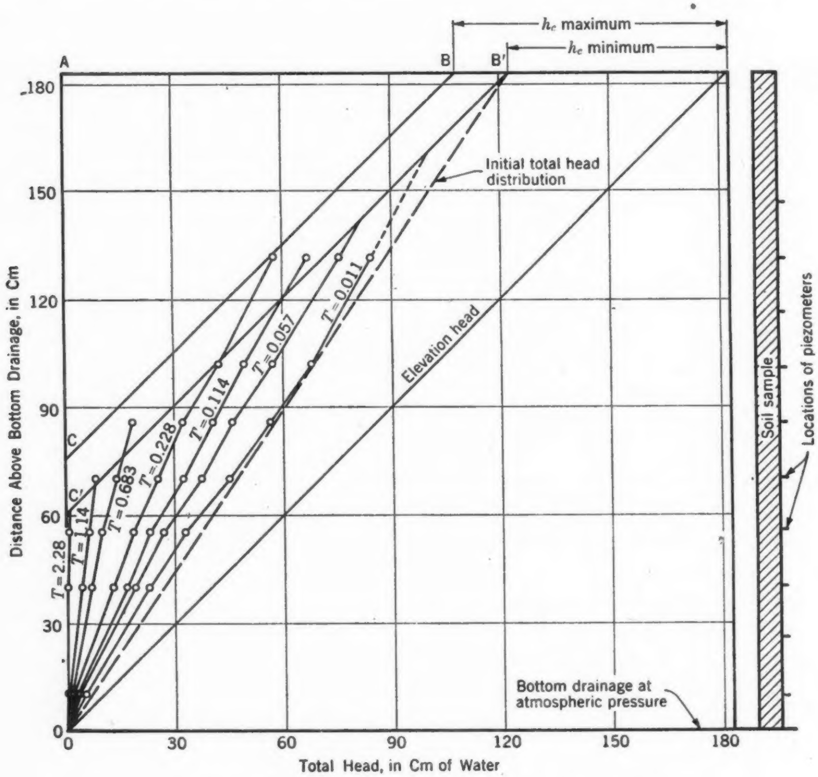

FIG. 16.—PORE PRESSURE DURING DRAINAGE OF SAMPLE

used in Figs. 14 and 15; T is the same as the author's τ (Eq. 23*a*), without the term 4. These total heads were obtained using a pore pressure measuring system with the same general principle as the one used by D. W. Taylor,[20] Assoc. M. ASCE, in his pore pressure work on undrained shear of clay. Since the pore water pressures were less than atmospheric pressure, it was necessary to prove that the measuring device correctly obtained the true pressures. This proof was secured by measuring the pore pressures for systems in which the values of pressure could be checked by an independent method.

[20] "Shear Research," 10th Rept. to U. S. Engr. Dept., Massachusetts Institute of Technology Soil Mechanics Laboratory, Cambridge, Mass., May, 1944.

If, at the start of the test, the level of the water is an infinitely small distance above the top surface of soil in the tube, the total water head at every point is equal to its elevation head as shown in Fig. 16. As soon as the water level drops below the top of the soil, menisci are developed, causing the soil to exert a tension (based on a zero pressure of atmospheric pressure) to the water. This tension is transmitted throughout the water and gives the initial total head distribution shown in Fig. 16. As drainage takes place, the total head at all points becomes smaller; at any time the total head distribution has a characteristic shape, as shown in Fig. 16, of smaller gradients at the lower elevations. When the water tension in any small element exceeds the capillary head, h_c, more water is removed than flows into the element and the degree of saturation becomes less than 100%. In Fig. 16, the line BC, or the line B′C′, represents the maximum water tension that can exist at any point. Thus,

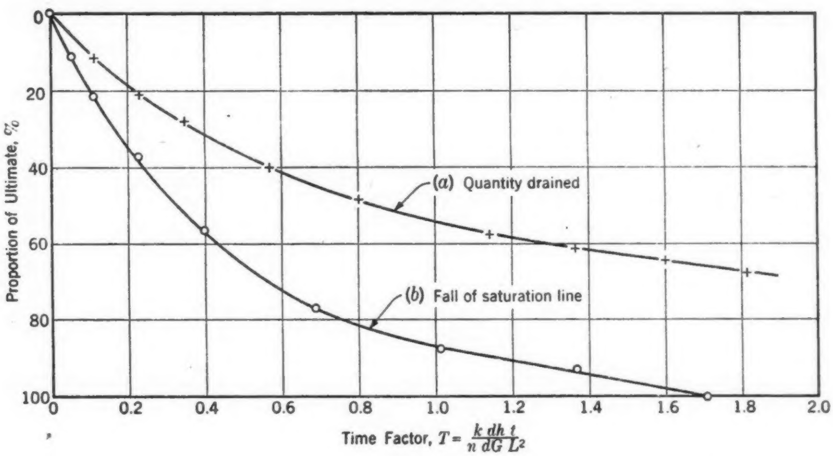

FIG. 17

at any time factor, line B′C′ is the boundary between the partially saturated soil above, and the completely saturated soil below, the line. Drainage from the partially saturated zone, represented by AB′C′, is a large percentage of the drainage and proceeds at a very slow rate because of the greatly lowered permeability.

The time lag of drainage from the partially saturated zone can also be noted in Fig. 17, which is a plot of the percentage of drainage and the relative fall of the line of "saturation" versus T for the laboratory test. The line of "saturation" fall was obtained by observation of the boundary line between the completely saturated and the partly saturated zones. There is some air present just below the visual boundary line, but this has little effect on the plot of fall of line of saturation since it is put on a percentage basis. Fig. 17 shows that the line of saturation has fallen to its final position when only 66% of the drainage has taken place, thus 34% of all drainage takes place from the partly saturated zone after the line of saturation has reached its final position; also,

while the line of saturation is falling, a large percentage of the water comes from above the line of saturation.

Although the writer feels that the use of analogies is very helpful for instructional purposes, he thinks that true derivations of analytical methods are desirable in order to clarify the various assumptions that are made. Therefore, a derivation of Eq. 4 is given. The general hydrodynamic equation governing laminar flow in soils is set up first. To save space, only one-dimensional flow

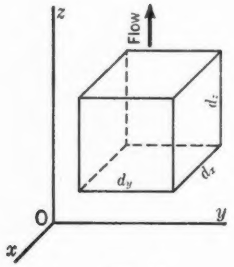

FIG. 18

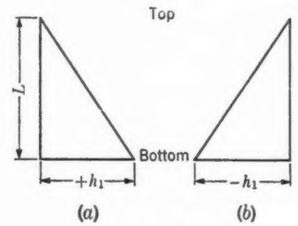

FIG. 19

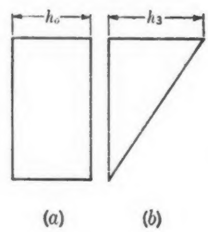

FIG. 20

is considered: The general equation can easily be extended to cover three-dimensional flow. The net flow into the element $dx\, dy\, dz$ (Fig. 18) will be the flow into the bottom face minus the flow out of the top face; that is,

$$\text{Net } q = \Delta q = q_{\text{bottom}} - q_{\text{top}} \quad \ldots\ldots(49a)$$

in which, by Darcy's law,

$$q_{\text{bottom}} = k_{\text{bottom}}\, i_{\text{bottom}}\, A = k_z \left(- \frac{\partial h}{\partial z} \right) dx\, dy \quad \ldots\ldots(49b)$$

and

$$q_{\text{top}} = k_{\text{top}}\, i_{\text{top}}\, A = \left(k_z + \frac{\partial k_z}{\partial z} dz \right)\left(- \frac{\partial h}{\partial z} - \frac{\partial^2 h}{\partial z^2} dz \right) dx\, dy \quad \ldots(49c)$$

In these equations q is the rate of flow; k is the coefficient of permeability; A is the cross-sectional area of soil; h is the total head; and i is the hydraulic gradient.

Substituting Eqs. 49*b* and 49*c* in Eq. 49*a* and combining terms,

$$\Delta q = \left(k_z \frac{\partial^2 h}{\partial z^2} + \frac{\partial k_z\, \partial h}{\partial z^2} + \frac{\partial k_z}{\partial z} dz \frac{\partial^2 h}{\partial z^2} \right) dx\, dy\, dz \quad \ldots\ldots(50)$$

In the element $dx\, dy\, dz$, the volume of water present is

$$V_w = \frac{G\, e}{1 + e} dx\, dy\, dz \quad \ldots\ldots(51)$$

in which G is the degree of saturation; and e is the void ratio. Therefore,

$$\Delta q = \frac{\partial}{\partial t}\left(\frac{G\, e\, dx\, dy\, dz}{1 + e} \right) = \frac{dx\, dy\, dz}{1 + e} \frac{\partial}{\partial t} (G\, e) \quad \ldots\ldots(52a)$$

or

$$\Delta q = \left(e \frac{\partial G}{\partial t} + G \frac{\partial e}{\partial t} \right) \frac{dx\, dy\, dz}{1 + e} \quad \text{(52b)}$$

Equating Eqs. 50 and 52*b*,

$$\left(k_z \frac{\partial^2 h}{\partial z^2} + \frac{\partial k_z\, \partial h}{\partial z^2} + \frac{\partial k_z}{\partial z} dz \frac{\partial^2 h}{\partial z^2} \right) dx\, dy\, dz = \frac{dx\, dy\, dz}{1 + e} \left(e \frac{\partial G}{\partial t} + G \frac{\partial e}{\partial t} \right) \quad \text{(53a)}$$

or

$$k_z \frac{\partial^2 h}{\partial z^2} + \frac{\partial k_z\, \partial h}{\partial z^2} + \frac{\partial k_z}{\partial z} dz \frac{\partial^2 h}{\partial z^2} = \frac{1}{1 + e} \left(e \frac{\partial G}{\partial t} + G \frac{\partial e}{\partial t} \right) \quad \text{(53b)}$$

—which is the general hydrodynamic equation for laminar flow in soils. By making various assumptions, several of the familiar formulas of soil mechanics can be obtained, for example:

(*a*) If it is assumed that k is constant and $G = 100\%$, Eq. 53*b* becomes

$$k_z \frac{\partial^2 h}{\partial z^2} = \frac{1}{1 + e} \frac{\partial e}{\partial t} \quad \text{(54)}$$

—which is the consolidation equation.

(*b*) If it is assumed that k, e, and G are constant, Eq. 53*b* becomes

$$k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{(55)}$$

—which is the Laplace equation for one-dimensional steady flow.

To obtain the author's formula (Eq. 4) it must be assumed that k is constant, e is constant, and

$$\frac{e}{1 + e} \frac{\partial G}{\partial t} = w_d \frac{\partial h}{\partial t} \quad \text{(56a)}$$

Assuming the relation expressed by Eq. 56*a*,

$$w_d = \frac{e}{1 + e} \frac{\partial G}{\partial h} = n \frac{\partial G}{\partial h} \quad \text{(56b)}$$

in which n is the porosity of the soil. Eq. 56*b* agrees in principle with the author's paper (under the heading, "Analogy Between Seepage and Heat Flow"), since $n\, \partial G$ is the water drained per unit volume. The writer prefers to use $n \frac{\partial G}{\partial h}$ instead of w_d, as both n and G are widely used terms in soil mechanics literature. If a vertical tube of soil is drained and the degree of saturation, G_z, of the drained soil is obtained at various heights above the atmospheric pressure line, a plot of z versus $n\,(1 - G_z)$ can be made. This is the true curve the author is attempting to plot in Fig. 6. There is some question as to the use of air to obtain the various hydrostatic heads, since it is difficult to know whether or not all the air pressure is carried by the water and whether or not the air picks up water in passing through the sample. Air pressure to obtain plots of $n\,(1 - G_z)$, which is also known as "effective porosity," versus z has been

used by other experimenters.[21,22] As recognized by the author, $n \frac{\partial G}{\partial h}$ (or w_d) may be far from constant.

The writer would like to point out that w_d and c_d have units. It would be helpful if the units were given to the values of w_d and c_d listed in the section, "Comparison of Computed and Observed Data," so as to enable the procedure to be better followed. For example, without units, the writer finds it difficult to discover how, by using Eq. 8*b*, the author obtains $c_d = \frac{0.8 \text{ in. per min}}{0.23} = 41.7$.

Making the three assumptions previously mentioned as necessary to obtain Eq. 4, by replacing $\frac{k_z}{n} \frac{\partial h}{\partial G}$ by c_d in Eq. 53*b*,

$$c_d \frac{\partial^2 h}{\partial z^2} = \frac{\partial h}{\partial t} \qquad (57)$$

—which is the author's equation (Eq. 21*a*) for vertical drainage.

Formulas for draining a saturated soil mass vertically through the base are next developed—because it is this flow condition for which the comparison is presented in Figs. 14 and 15.

The solution of Eq. 57 involves the assumption of an initial distribution of total head, h. As in the theory of consolidation for the initial distribution of hydrostatic excess, there are many initial total head distributions for which solutions can be obtained. By using boundary conditions of bottom drainage only and of zero total head at the bottom immediately after start of flow, by assuming an initial total head distribution as shown in either Fig. 19(*a*) or in Fig. 19(*b*), and by solving in a manner similar to the solution for the consolidation theory,[23]

$$d = 1 - \frac{16}{\pi^3} \sum_{n=1}^{n=\infty} \frac{(2\,n - 1)\,\pi + 2\,(-1)^n}{(2\,n - 1)^3} e^{-\frac{1}{4}\lambda\tau} \qquad (58)$$

—which is Eq. 25*a*, with a correction of 4 in the exponential term. The boundary conditions of bottom drainage and of zero total head at the bottom are in complete agreement with what takes place. On the other hand, the assumption of either initial total head distribution as shown in Fig. 19 is questionable. A solution based on Fig. 19(*a*) gives an infinite initial gradient at the bottom; also, according to this solution, during the early stages of drainage there is upward flow in the upper sections of the soil. Examination of Fig. 16 shows that both these conditions are untrue. A solution based on Fig. 19(*b*) gives water flow into the soil sample and, therefore, has no place in drainage equations.

It is interesting to note that the solution of Eq. 58 can be obtained by substitution of the initial condition of Fig. 19 in Eq. 57, or by correctly combining the better known solutions for initial conditions as shown in Fig. 20.[23] In view of the actual initial total head distribution in Fig. 16, the writer suggests that a solution based on an initial total head distribution as shown in Fig. 20(*b*), in which $h_3 = L - h_c$, is superior to that presented by the author. This solution

[21] "Capillarity of Soils," by Hsien-Hsiang Ku, thesis presented to Purdue University at Lafayette, Ind., in 1940, in partial fulfilment of the requirements for the degree of Bachelor of Science.

[22] "Subsurface Drainage Investigation," New England Div., Corps of Engrs., Dept. of the Army, Boston, Mass., 1947.

[23] "Fundamentals of Soil Mechanics," by D. W. Taylor, John Wiley & Sons, Inc., New York, N. Y., 1948.

is given by K. Terzaghi and O. K. Fröhlich[24] and will be called theory C for the comparison in Figs. 14 and 15. The time curves for theory C are given in Fig. 21. A comparison of the theoretical time curves of Fig. 21 with the actual time curves of Fig. 16 shows: (*a*) The theoretical initial total head distribution

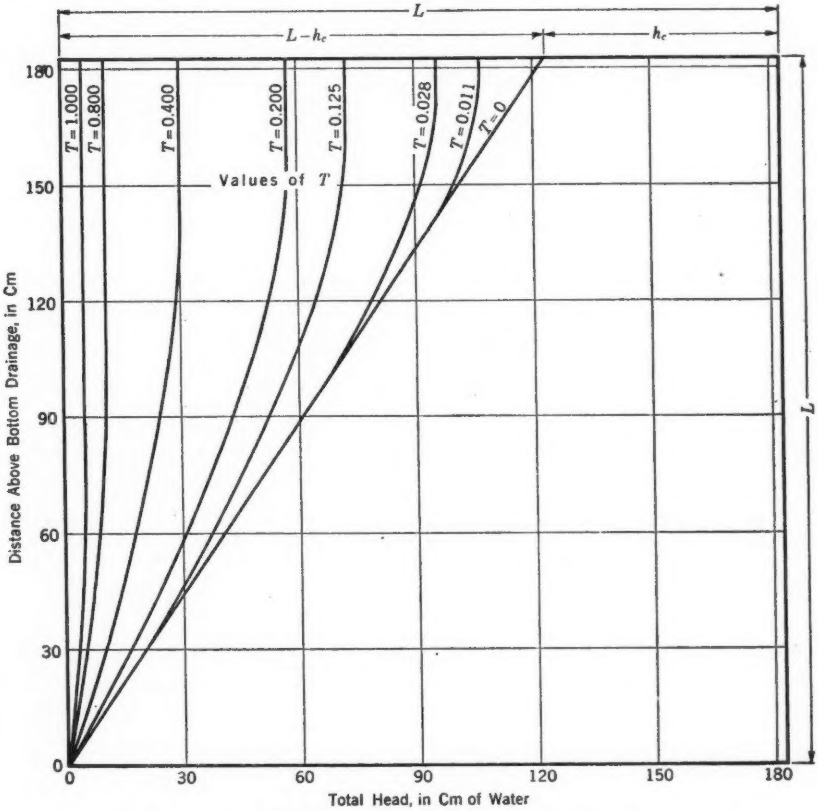

FIG. 21.—TOTAL HEAD AS A FUNCTION OF HEIGHT AND TIME FACTOR

is correct; (*b*) in the early stages, the actual head dissipation is faster than in theory C (later the reverse is true); and (*c*) the theory does not take into account the partly saturated zone. As one might expect, theory C agrees fairly well with the actual test, as noted in Fig. 14, until the effects of the partly saturated zone become important.

The ultimate total water drained can be computed for theory C by: (1) Setting up the expression for total head as a function of time and distance; (2) differentiating the head expression with respect to z to obtain the gradient and substituting $z = 0$ to obtain the gradient at the bottom; and (3) putting the gradient in Darcy's expression for rate of flow and integrating with respect to time from time $= 0$ to time $= \infty$. For the case studied, the ultimate total water obtained by this method was only 57% of the actual. This result is

[24] "Theorie der setzung von tonschichten, eine einführung in die analytische tonmechanik," by K. Terzaghi and O. K. Fröhlich, Deuticke, Leipsig, 1936.

not surprising in view of the fact that the theory does not approximate the actual head dissipation in the partly saturated zone. The ultimate quantity drained was not computed using the author's initial head distribution because the gradients obtained have little meaning. Since the bottom gradient is infinite immediately after starting drainage, a solution, like that applied to theory C, using the author's initial total head distribution, would give an infinite quantity. It would be helpful if the author were to explain how the ultimate quantity can be computed in a manner consistent with his theory, if such a method exists.

The remaining two theoretical solutions plotted in Fig. 14 also require discussion. In his theory, K. Terzaghi assumes that all drained water comes from a small cross-sectional element of soil just below the line of saturation, thus neglecting all flow from the partly saturated zone. Fig. 17 shows that this assumption may be far from correct. K. Terzaghi recognized this fact himself and in his book[25] states that this method may be only a "* * * very crude approximation * * *." In applying this theory, the value of capillary head, h_c, must be known. The writer has not used the definition of h_c given by K. Terzaghi, but he has selected values of the varying h_c from test data. More specifically, the value of h_c used in the gradient expression is the value required to give the actual initial gradient. (This value of h_c was also used for theory C.) The capillary head and the final value of G chosen to compute the quantity were average values so that $A\, n\, (1 - G)\, (H - h_c)$ equaled the measured quantity drained. Such selections of values of G and h_c would not be possible had actual drainage test results not been available; they were made so that the comparison in Fig. 14 would be on a more consistent basis.

The transient flow net solution is based on the assumption that at any instant the flow can be represented by a steady state flow net. The solution completely neglects the effect of capillarity and, thus, for the case of draining a vertical column of soil from the bottom, assumes a constant gradient of unity. It is not surprising, therefore, that this solution gives a much too large rate of seepage. The difference between this solution and the actual would be even larger for a finer-grained soil where the capillary effects would be more pronounced.

In conclusion, the writer would like to emphasize that the author's method, as well as the other theoretical methods considered in this discussion, does not recognize the changing permeability that appears in Eq. 53*b* and which must be considered in draining soils with large capillary heads. Also, all the methods assume that the void content of the soil does not change under the added intergranular load caused by drainage. In fine-grained soils this assumption of constant void content may be questionable.

Corrections for *Transactions:* In September, 1947, *Proceedings*, page 1068, line 10, change "$u_w\, \gamma$" to $\frac{u_w}{\gamma}$, and Eq. 1 should read $h = z_p - z_d + \frac{u_w}{\gamma}$"; and, on page 1075, Fig. 4, delete two straight curves with their corresponding plotted points.

[25] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, p. 310.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF STEPPED-COLUMN MILL BENTS

Discussion

By E. I. Fiesenheiser, Walter J. Gray, L. J. Mensch, and Arthur R. Graves

E. I. Fiesenheiser,[7] M. ASCE.—The use of moment distribution and the balancing of shear produced by sidesway provide a convenient solution for stresses in the mill building bent. An alternative method for determining the fixed-end moments in the stepped column involves only shear and moment dis-

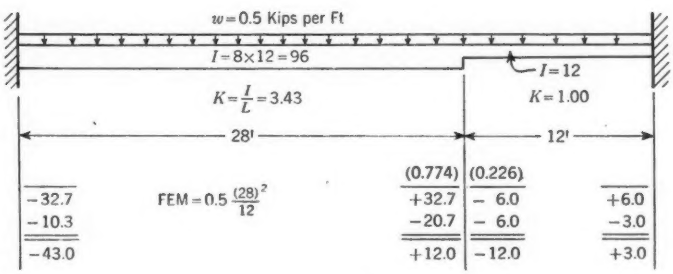

Fig. 17

tribution for the stepped member itself. This method is convenient and does not require the use of moment diagrams. Since it may be of interest to some engineers, it is presented herein.

The procedure is to assume the stepped column to be a combination of two separate members. Fixed-end moments due to loads on the separate members are first computed and balanced, and the resulting unbalanced shear (or joint restraint), acting at the junction of the two members, is then calculated. The shear is balanced by application of an equal and opposite force at this joint. In calculating the effect of the applied force the joint is allowed to deflect without rotation. In this case the applied moments must be proportional to

Note.—This paper by Daniel S. Ling was published in October, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1947, by Ralph E. Spaulding.

[7] Associate Prof. of Civ. Eng., Illinois Inst. of Technology, and Consultant, Inst. for Nuclear Studies, Univ. of Chicago, Chicago, Ill.

I/L^2 for the two segments of the stepped column and then balanced. Shears are computed and balanced by use of the proper correction factor. To obtain the final fixed-end moments, the moments caused by the loads acting on the

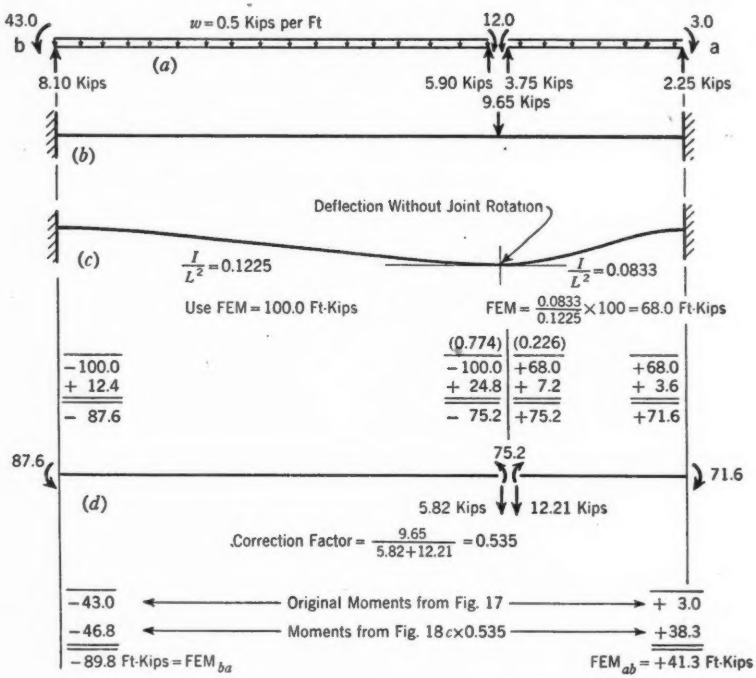

FIG. 18

separate members and those due to the shear balance are added.

The alternative method is illustrated by the calculations shown in Figs. 17, 18, 19, and 20. These calculations are made for the problem in Example 1, in

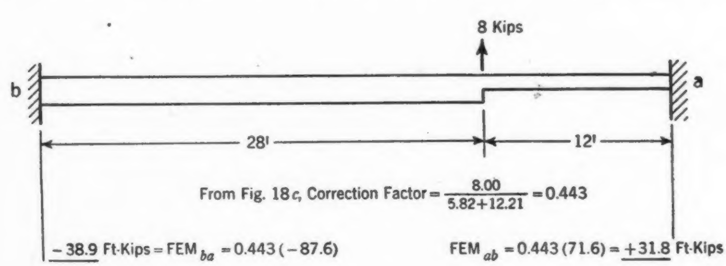

FIG. 19

which the ratio of moments of inertia is given as 8 to 1. Since only relative values are significant, moments of inertia of 96 and 12 are assumed. Figs. 17 and 18 show the calculation of the fixed-end moments for a wind load of 0.5

kips per ft on the 40-ft column. Fig. 19 shows the fixed-end moment computation for the effect of the 8-kip force due to crane thrust; and Fig. 20, the effect of the applied moment of 120 ft-kips at the joint.

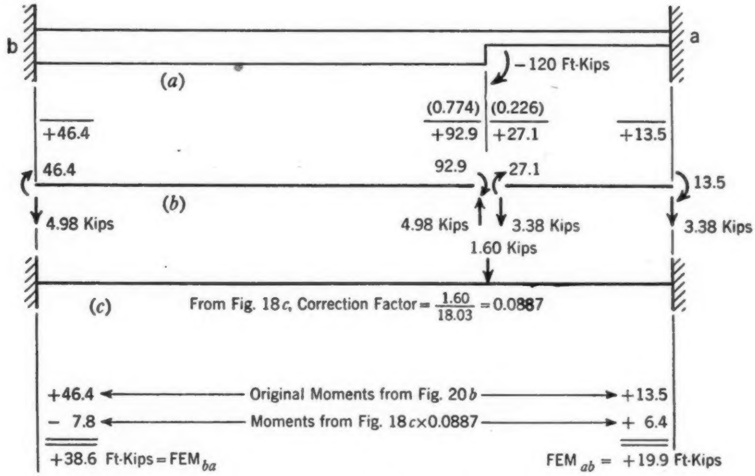

FIG. 20

It is noted that the fixed-end moments calculated by this method compare reasonably well with those calculated by the author. The use of moment diagrams and the procedure of dividing the member into many segments is unnecessary.

WALTER J. GRAY,[8] JUN. ASCE.—The sets of factors presented in this paper permit rapid analysis of the examples. However, they are not sufficiently complete for general use. Additional curves should be presented to show:

1. Fixed-end moments for concentrated side loads at other positions than at crane level, permitting a direct solution of the case of shed roof thrust as shown in Fig. 12.
2. Distribution factors, permitting a direct evaluation of the effects of truss restraint and lower floor restraint.
3. Fixed-end moments, indicating the effects of eccentric application of loads on the column, although these might be approximated from fixed-end moments for concentrated side loads at positions other than at crane level.

The carry-over factor curves should be redrawn to give a direct reading in one set of curves, thus avoiding the use of two factors. Fig. 21 is such a set of direct reading carry-over factor curves with a limiting value of $I/I_o = 10$, as given in an unpublished paper written by the writer in 1942 on "Analysis of Stepped Crane Columns by Moment Distribution," which is on file in the

[8] Civ. Engr., Public Utilities Comm., City of San Francisco, San Francisco, Calif.

Engineering Societies Library at New York, N. Y. Values were calculated by the column analogy[9] procedure of Hardy Cross, Hon. M. ASCE.

The design of the stepped column is not complete, however, even though the combined stresses due to maximum moments and thrusts are computed,

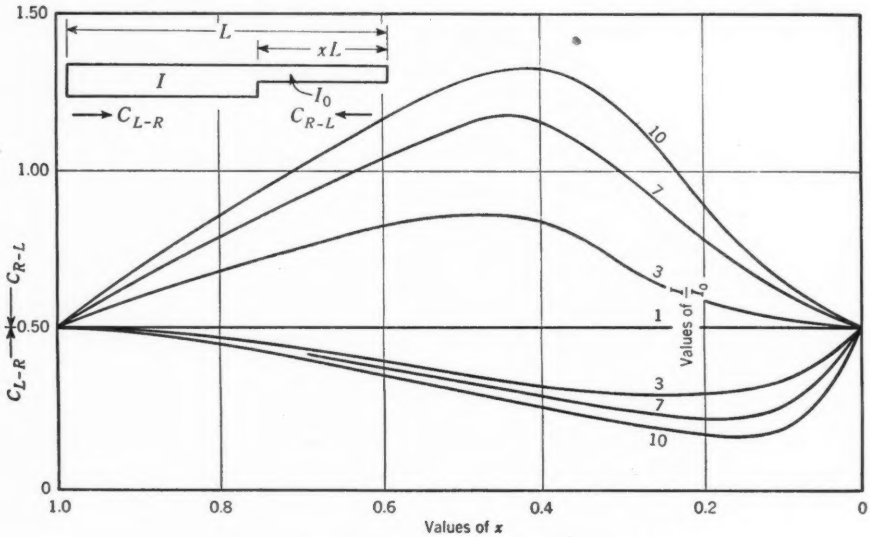

FIG. 21.—CARRY-OVER FACTORS FOR STEPPED MEMBER

since they must be compared with the allowable combined stress in a stepped column. This latter value requires consideration of the slenderness ratio of the column before an allowable axial stress value can be determined. Thomas C. Shedd,[10] M. ASCE, suggests using an "equivalent" slenderness ratio of $L_1/R_1 + L_2/R_2$.

L. J. MENSCH,[11] M. ASCE.—There was a dearth of technical literature on the analysis of mill bents by the theory of elasticity until about 1913 when a treatise appeared on the subject[12] in full panoply by W. Gehler, which showed the analysis of each of the bents in Fig. 22 for nearly ten loading conditions. Simple working formulas, clearly arranged, were given for each case—formulas not equaled by later authors in the ease of arrangement. This enabled engineers to design such bents in a very short time. A number of writers refigured some of the cases.[13,14,15] Had the author compared his results

[9] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., October, 1930.

[10] "Structural Design in Steel," by Thomas C. Shedd, John Wiley & Sons, Inc., New York, N. Y., 1934, p. 395.

[11] Gen. Contractor, Evanston, Ill.

[12] "Der Rahmen," by W. Gehler, W. Ernst und Sohn, Berlin, 1913.

[13] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, *Bulletin No. 108*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1918.

[14] "The Structural Engineer's Handbook; Data for the Design and Construction of Steel Bridges and Buildings," by Miles S. Ketchum, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1924.

[15] "Rahmen Formeln," by Adolf Kleinlogel, W. Ernst und Sohn, Berlin, Vol. 8, 1939.

with any of those in the references suggested, he would have found his assumptions incorrect, as the moments at the foot of the columns are only one half the correct amount.

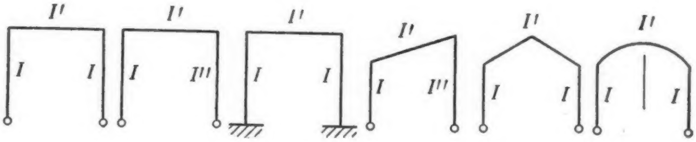

FIG. 22

It is true that no formulas are available for bents with stepped columns and that the theory of elasticity would require the solution of a structure three times indeterminate complicated by the presence of a roof truss; this is a task even beyond the capacity of a large engineering organization, when it has to design perhaps five bents each for five loading conditions.

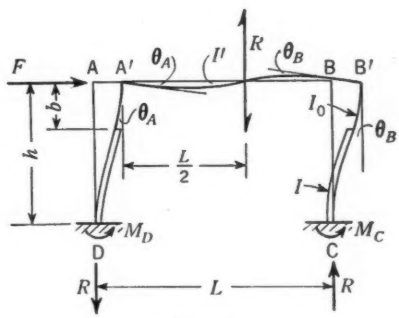

FIG. 23

Many years ago the writer stumbled on a solution that gives values within a small percentage of the theoretical results. It is based on the fact that the truss is from two times to ten times stiffer than the columns, and on the fundamental loading condition by a horizontal force at the top of one column. A symmetrical bent will deform as shown in Fig. 23. By inspection,

$$\theta_A = \theta_B \quad (11)$$

$$M_A = R\,\frac{L}{2} \quad (12)$$

$$AA' = BB' \quad (13)$$

$$M_D = M_C = h\,\frac{F}{2} - R\,\frac{L}{2} = \frac{F\,h}{2} - M_A \quad (14)$$

and

$$F\,h = M_D + M_C + R\,L = 2\,M_D + R\,L \quad (15)$$

From Eqs. 14 and 15 it will be seen that if R or M_A were known, all other unknown quantities could be computed. This can be effected by expressing θ_A twice—once by the deformation of the truss, and once by the deformation of column AD.

Assuming the truss to have parallel chords and the moment of inertia, I',

$$\theta_A = \frac{R\,L^2}{12\,E\,I'} = \frac{M_A\,L}{6\,E\,I'} \quad (16)$$

From the column deformation for a uniform column section,

$$\theta_A = \int_0^h \frac{M\,dx}{E\,I} = \frac{\frac{x\,F}{2}\,dx - M_A\,dx}{E\,I} = \frac{\frac{F\,h^2}{4} - M_A\,h}{E\,I} \qquad (17a)$$

For stepped columns the column deformation is

$$\theta_A = \int_0^b \frac{\frac{F}{2}\,x\,dx - M_A\,dx}{E\,I_0} + \int_b^h \frac{\frac{F}{2}\,x\,dx - M_A\,dx}{E\,I} \qquad (17b)$$

Four particular combinations of values for stepped columns will be considered, as follows:

Combination	Values of I/I_0	Values of b
1	5	$h/3$
2	5	$h/4$
3	10	$h/3$
4	10	$h/4$

For these combinations, respectively,

$$\theta_A = \frac{13}{36}\frac{F\,h^2}{E\,I} - \frac{7}{3}\frac{M_A\,h}{E\,I} \qquad (18a)$$

$$\theta_A = \frac{10}{32}\frac{F\,h^2}{E\,I} - 2\,\frac{M_A\,h}{E\,I} \qquad (18b)$$

$$\theta_A = \frac{1}{2}\frac{F\,h^2}{E\,I} - 4\,\frac{M_A\,h}{E\,I} \qquad (18c)$$

and

$$\theta_A = \frac{25}{64}\frac{F\,h^2}{E\,I} - \frac{13}{4}\frac{M_A\,h}{E\,I} \qquad (18d)$$

Denoting $I'/L \div I/h$ by n, and eliminating θ_A by combining Eq. 16 with Eqs. 17*a* and 18, for a column of uniform section,

$$M_A = \frac{F\,h}{4}\left(\frac{6\,n}{1 + 6\,n}\right) = 0.242\,F\,h \qquad (19)$$

For the four combinations of I/I_0 and b previously given,

$$M_A = \frac{13}{84}\,F\,h\left(\frac{14\,n}{1 + 14\,n}\right) = 0.1525\,F\,h \qquad (20a)$$

$$M_A = \frac{5}{32}\,F\,h\left(\frac{12\,n}{1 + 12\,n}\right) = 0.1535\,F\,h \qquad (20b)$$

$$M_A = \frac{1}{8}\,F\,h\left(\frac{24\,n}{1 + 24\,n}\right) = 0.124\,F\,h \qquad (20c)$$

and

$$M_A = \frac{25}{208}\,F\,h\left(\frac{19.5\,n}{1 + 19.5\,n}\right) = 0.1185\,F\,h \qquad (20d)$$

Inasmuch as n generally has a value of 5 or more, the fraction containing n in Eqs. 19 and 20 is nearly equal to 1; the approximate values were determined for $n = 5$. By Eq. 14, it can easily be proved that, for uniform columns,

$$M_D = 0.258\,F\,h \qquad (21)$$

and for the four combinations of I/I_0 and b previously given,

$$M_D = 0.3475\,F\,h \qquad (22a)$$

$$M_D = 0.3465\,F\,h \qquad (22b)$$

$$M_D = 0.376\,F\,h \qquad (22c)$$

and

$$M_D = 0.3815\,F\,h \qquad (22d)$$

The results given by Eqs. 21 and 22 enable solution for other loading conditions, as follows:

Assume a column of uniform section to be affected by a horizontal wind load, w, per linear foot. A relaxation type method is used. If the column is not only fixed at point D, but is also held fixed at the top of point A, the end moments are $w\,h^2/12$ and the end shears are $w\,h/2$. If point A is released, the shear $w\,h/2$ will act like the force F in the fundamental case and

$$M_D = \frac{w\,h^2}{12} + 0.258\,f\,h = \frac{w\,h^2}{12} + 0.129\,w\,h^2 = 0.212\,w\,h^2 \qquad (23)$$

The shear at the bottom of the column is

$$V = \frac{w\,h}{2} + \frac{F}{2} = \frac{3}{4}\,w\,h \qquad (24)$$

In the case of a stepped column, for example, when $I/I_0 = 10$ and $b = h/3$, the first step is to compute the moments at the ends of the column, considered as a beam uniformly loaded (say, by the slope-deflection method), from which

$$M_D = 0.123\,w\,h^2 \qquad (25a)$$

and

$$M_A = 0.052\,w\,h^2 \qquad (25b)$$

and the shears at points D and A are

$$V_D = \frac{4}{7}\,w\,h \qquad (26a)$$

and

$$V_A = \frac{3}{7}\,w\,h \qquad (26b)$$

The final values at point D are

$$M_D = 0.123\,w\,h^2 + (0.376 \times w\,h^2)\,\frac{3}{7} = 0.284\,w\,h^2 \qquad (27a)$$

and

$$V_D = \frac{4\,w\,h}{7} + \frac{3\,w\,h}{14} = \frac{11\,w\,h}{14} \qquad (27b)$$

Similarly, for $I/I_0 = 5$, and $b = h/3$,

$$M_D = 0.277\,w\,h^2 \qquad (28a)$$

and

$$V_D = 0.77\,w\,h \qquad (28b)$$

Other loading conditions may be found by the same short cut. For uniform vertical loads on the truss, the point of inflection of the columns will be found to be: For $I/I_0 = 5$, between $0.04\,h$ and $0.06\,h$ below the midheight; for $I/I_0 = 10$, between $0.06\,h$ and $0.02\,h$ above the midpoint; and for a column of uniform section, $h/3$ above point D.

It is assumed in the foregoing analyses that the foot of the column is fixed. In many bents the columns rest on walls or piers, often 20 ft high. These supports are affected by the moments M_D and by the shears, and must be designed accordingly.

There is a question as to what value of I' to use for a Fink truss of a depth equal to perhaps one fifth of the span. The moment of inertia will be greater than that of a parallel chord truss having a depth of about one tenth that of the span; and the foregoing formulas can be used safely, as long as a stiff connection is provided between truss and columns by a brace capable of transferring the moments occurring at the top of the columns.

ARTHUR R. GRAVES,[16] M. ASCE.—The chief contribution of this paper to engineering and technical knowledge lies in Figs. 6, 7, 8, and 9, which may be used in calculating the fixed-end moments for three kinds of loading, and the distribution coefficients and carry-over ratios for any kind of loading. All, of course, are restricted to members with only one abrupt change of section, but their use is not limited to the design of stepped columns. The mill building designer needs such timesaving devices as these because as soon as a project is decided on, the owner, thinking of his anticipated profits, considers every day a loss of one day's profits until the mill is working—the larger the mill, the greater is the urge for speedy construction.

The foundations must be started first and this cannot be done until the column bents are designed. For columns with a constant moment of inertia, the points of contraflexure may be assumed and the horizontal shears distributed over the columns with comparatively little work, yet with satisfactory results. For stepped columns, however, the results from this approximate method will, in most cases, be far from those obtained by more exact methods. An indication of how serious these variations can be is shown by the range in values of the various functions given in Figs. 6, 7, 8, and 9. Another more convincing demonstration may be seen in some of the buildings in which the columns must be plumbed, or in which the crane runway must be lined up every few years, because the actual moments on the base of the columns are much more than those calculated by the approximate method.

In most cases, a more exact analysis of the column bent takes so long that the designer cannot delay the foundations long enough to make it. With the use of these or similar charts (Figs. 6, 7, 8, and 9) the labor and the time of calculation are reduced so that it is practicable to make a closer analysis of the typical bents, which, in turn, afford a better base on which to approximate the irregular bents, if time does not permit a closer analysis of them. It is possible for each designer to make up his own charts, but it is not practicable because of the time and labor required. The writer has a different set of charts, serving the same general purpose, which were developed over a period of more than ten years.

Even with charts such as the author's (Figs. 6, 7, 8, and 9), there is still much work to be done in analyzing column bents, and for most common mill bents the method outlined in this paper may be shortened without too great an inaccuracy in the final results. This method takes into account the deflec-

[16] Senior Designer, Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

tion of the column between the knee braces and the roof truss. In the most common mill bents this refinement is not as important as it seems at first. The chief effect of this deflection of the column is that it makes the bent more limber and increases the horizontal deflection. If this deflection is assumed not to exist, two more charts may be plotted which will further reduce the amount of work. One chart should locate the point of contraflexure of the bent under translation forces. The other chart should give a stiffness factor to be used in distributing the translation forces among the columns in the bent.

With this alternate method the fixed-end moment at the bottom of the column will be less than that obtained by the original method outlined in the paper. The moment from translation will be greater, however, and, when the fixed-end moment and the translation moment are added together, the resulting moments from the two methods will differ by a small percentage.

The fixed-end moment at the top of the column will be greater with the alternate method and so will the moment from translation for wind load and crane thrust. However, these moments are always opposite in sign and their difference is changed much less than either of the two moments. The fact that the fixed-end moment and the translation moment at the top of the column are opposite in sign makes it possible to vary the calculated moment at this point, due to crane thrust of eccentric load, by varying the number of bents over which the translation effect is assumed to be distributed. With cranes having heavy trolleys the forces involved are considerable. In this connection it is also possible to have the unloaded columns of a bent of just the right stiffness so that the translation moment is equal and opposite in sign to the fixed-end moment at the top of the loaded column. This would result in a calculated moment of zero. To take care of such possibilities it is advisable that no column be designed for less than 50% of the fixed-end moment at the top of the column.

In the case of eccentric loads the signs of the fixed-end moments at both top and bottom of the column will vary with different ratios of the moments of inertia and with different ratios of the length of the upper part of the column to the entire length of the column. For one column the fixed-end moments may have the same sign as the translation moment; for the next column the reverse may be true. This variation in the fixed-end moment is clearly shown in Fig. 9. When adding fixed-end moments from eccentric loads to the translation moments the sign of each moment should be carefully checked.

It should be noted that the author's charts, given in Figs. 6, 7, 8, and 9, were published previously by L. C. Maugh,[2] M. ASCE.

Corrections for *Transactions:* In October, 1947, *Proceedings*, on page 1276, rotate Fig. 4, counterclockwise 90°; on page 1277, line 1, change "point b" to "point a"; in Fig. 5(*a*), enterchange the joint designations "a" and "b"; in Table 1 (in the title) change "Point b" to "Point a" and (in the first column) change "bc" to "aa"; and, on page 1284 ,in the second line of the last paragraph of Section IV, delete "by" and substitute "to problems in."

[2] "Statically Indeterminate Structures," by L. C. Maugh, John Wiley & Sons, Inc., New York, N. Y., 1946.

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DISCUSSIONS

SUBSIDENCE OF THE TERMINAL ISLAND-LONG BEACH AREA, CALIFORNIA

Discussion

BY JACOB FELD, AND HYDE FORBES

JACOB FELD,[6] M. ASCE.—There are so few authentic data concerning the extent and cause of surface subsidence resulting from liquid removal from the subsoil that this paper should be read with interest and studied with care. From a qualitative point of view, the authors undoubtedly show the cause of the major portion of the recorded subsidence; from a quantitative approach, the methods used do leave "much to be desired," and cannot become a model for the solution of similar problems.

A comparison of the contours when drawn to the same scale shows: In Fig. 1, the actual subsidence measured from 1933 to 1945; in Fig. 11, the calculated subsidence as of 1945; and, in Fig. 12, the probable ultimate subsidence.

This comparison indicates the following general observations:

1. The ultimate subsidence will be about 175% of the amounts measured in 1945; and
2. There is no apparent correlation between the data in Figs. 1 and 11, and the pattern of the predicated ultimate subsidence bears no relation to that of the computed values.

These conclusions are not consistent with the information given in other parts of the paper. The shape of the settlement time curves of the four bench marks shown in Fig. 3 does not indicate a decreasing rate of settlement during recent years. Quite the contrary conclusion can be reached from the graphs in Fig. 2, as well as Fig. 3, since the first foot of settlement required about 5 years, the second foot and third foot each occurred in $1\frac{3}{4}$ years (average), and both the fourth foot and fifth foot occurred in about 1 year—all referred to 1933 as datum.

The chief cause of such a lack of coordination seems to be the implicit reliance on the method used for computing expected subsidence. Drilling cores, several years old, when tested in the laboratory were assumed to be

NOTE.—This paper by Frederic R. Harris and Eugene H. Harlow was published in October, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1948, by James Gilluly and U. S. Grant.

[6] Cons. Engr., New York, N. Y.

typical of geological zones, and were taken to have average thicknesses of 500 ft and 750 ft, respectively. The computations were corrected by multiplying the shale, sandstone, and other expected compressible layers by 8% (that is, a reduction in computed values by 92%) to make one spot agree with the measured settlement at that point. It might be more logical to assume that only one layer—the weakest—caused all the subsidence.

The discussion of actual and expected consolidation settlements from soil compression and from unwatering operations indicates values quite consistent with those measured in similar soil areas along the eastern seaboard. Filled areas covering salt marshes and silt where the soil water is affected by tides, although protected by tight bulkheads, showed settlements of about an inch for each 5 years. The area along the Harlem River in New York, N. Y., has been under observation for about 75 years, and the total subsidence in that period is more than 15 in. For nearly a year dewatering in this area to a depth of 35 ft (45 ft below ground level) caused a maximum, almost immediate, additional settlement of about 5 in. nearest the excavation, with no measurable settlement some 150 ft away. In another area of similar soil conditions about 1,000 ft from tidewater, a ground-water lowering of 45 ft (55 ft below street surface) settled the surface about 3 in. In both cases very little rebound of the surface was noted when the water table was permitted to rise to normal level.

The paper, as printed, is a summary of a much longer report which may contain more convincing proof that the quantative method used is reliable, and, if so, it is hoped that the authors will publish such proof in the closing discussion.

HYDE FORBES,[7] M. ASCE.—Subsidence is a phenomenon common to many localities in California, caused by the increase in ground-water pumping, since about 1918. In this presentation of the measurement of the phenomenon, the authors advance the conclusion that it is caused principally by the consolidation of shale members of the productive oil formations underlying the area. This latter view is open to question. The total subsidence shown in Fig. 12 is less than that known to have occurred during the same period at the southern end of San Francisco Bay (California), where there was no oil production.

The paper enumerates the factors contributing to the subsidence of the area in question (with which there is no disagreement) and then places a measure on each in turn:

(*a*) About 0.2 ft is attributable to surface loading;

(*b*) The effect of pumping water from the coastal plain southwest of the Newport-Inglewood fault is summarized (under the heading, "Factors Contributing to the Subsidence: Water Pumping for Industrial and Domestic Uses") as "the fraction of the total subsidence resulting from this source amounting to inches, not feet";

(*c*) The subsidence caused by the dewatering of the dock site to El. −75, based on the theoretical consolidation resulting, was assumed to be limited to the time of such dewatering and to cease with the measurement in the spring of 1942 (using curve (*e*) in Fig. 3, this would be less than 1 ft); and

[7] Cons. Engr. and Geologist, Palo Alto, Calif.

(*d*) The subsidence resulting from the consolidation of the oil measures to considerable depths because of the extractions of gases and fluid. Item (*d*) is cited as the major factor.

The writer will discuss these factors from the point of view of his knowledge of conditions, believing that the subsidence in the Terminal Island area, in particular, is due to three causes which, although incapable of exact measurement (but in order of magnitude), are: (1) The dewatering and the compacting of the sand making up the island and its continuation toward the shore below tide level; (2) the dewatering and the compacting of the alluvium of the Southern California coastal plain with the increased draft upon ground water for water supply purposes; and (3) possibly some compaction of the sands of the oil measures with the withdrawal of their fluid content under the weight of the overburden. It is doubtful that the shale members of the bedded sediments have suffered any further consolidation since 1937. It is true that the interbedded sands would provide drainage of facilities equivalent to the porous stones (of the tests performed). However, the increased loading required for consolidation of the shale does not exist, because the additional surface loading to be transmitted to depth is much less than the fluid load that has been removed from the overlying beds and sediments.

The writer's first acquaintance with Terminal Island was in the school year, 1907–1908, when the Saturday field trips of the Los Angeles (Calif.) High School science class took him to the island and to the adjacent Long Beach-San Pedro coast line. The lecture about the island was of great interest and of lasting memory. It was used, by the well-informed Prof. J. Z. Gilbert, to illustrate the formation and composition of a sand spit, which was a section of an extensive sand bar extending above water level. The bar had been built up, torn down, reformed, and shifted in position from time to time by the action of ocean shore currents and waves upon the coarser sediments brought to the shore line and dropped by both the Los Angeles and San Gabriel rivers. The spit and bar, with its overlying estuary mud, consisted of more than 50% water by volume, and so was unstable under the attack of tidal current and wave action.

The writer sees no reason to change the concept passed on to him at that time. The construction of the breakwater and the dredging of the harbor minimized the wave and current action and, with the filling in of the marshland between the island and the present location of the Pacific coast highway, the sand above and below the estuary and shore mud and clay settled to firmer bearing as the loosely held water was forced out of the pore space. At that time ground-water levels were well above high tide. Fresh water springs flowed into slough channels in the marsh at low tide. Some of these channels were dammed to provide for fresh water duck ponds, and there were numerous bodies of fresh water in slough channels cut off from tidewater.

With these facts in mind, it is difficult to agree with the authors that the filling and loading of that extensive marshland and island, more than 1 mile in average width, was not responsible for continued settlement and the more than 0.2 ft adopted for the period. The base of the cut and cover fill placed over garbage deposited along the shore of San Francisco Bay is continuing to settle with the compaction of the bay mud. Also, considering the extent and

physical make-up of the sand, it is unlikely that all settlement, due to dewatering those sands to El. −75 for dry dock construction, ceased with the cessation of pumping at the end of the construction period. Even though water levels in wells were shown to recover, that necessarily is not proof that the pore space between sand grains is not, although slowly, contracting further as the lesser amounts of water held in them rises to the former level registered in the wells.

The void space that comprises the storage capacity of a ground-water reservoir contracts with the withdrawal of water or with the lessening of hydrostatic pressure. When the water column again reaches its former level at the higher part of the reservoir or basin, equal hydrostatic pressures are transmitted throughout; but the void space or storage capacity is never recovered, constituent grains never being unseated. This has been clearly demonstrated in the Santa Clara Valley of California through the observation of well water levels and discharge and artesian flow from wells during the period from 1933 to 1948 (24).[7a] The same thing can be assumed to be true in relation to oil sands. When the oil migrated to the sand beds it occupied voids filled with water in which the sediments were deposited. The oil and gases (being lighter) rose in the water, forcing the water down the dip of the strata, and were compressed by the pressure so developed in the formation. As the volume of gas, oil, and water was removed from the sands, the void space would contract. Just when the subsidence due to dewatering the top sand to El. −75 ended (if it has), and just when the subsidence due to voiding the oil sands of their fluid became effective, is a matter for question, and, in the opinion of the writer, indeterminate.

During 1920–1921 the writer made a study of the Southern California coastal plain, from Redondo Beach (Calif.) to beyond Long Beach, for the Shell Oil Company to locate and determine the character of geologic structures buried by the alluvium of the plain through the application of ground-water hydrology. The study consisted of a canvass of all water wells of the area; a compilation of available information as to log, depth, water yield, and past water levels; and the determination and correlation of the then existing elevation of well water levels through lines of levels run by accompanying survey parties. Several facts had a bearing on the subsidence that was then occurring with the lowering of ground-water levels and the restriction of the artesian flow area with the increase of pumpage. The bench marks of the areas between Signal Hill, Dominquez, and Palo Verdes Hills and along the coast south and east of Signal Hill were consistently so much lower than their published datum that it was obvious that all minus determinations could not be attributed to accumulative error. Owners of dug wells along some sloughs volunteered the information that the water from those wells had been potable but, since the breakwater was built and the harbor was dredged, the tides changed and came so far up the sloughs that they seeped into their wells and made the water so brackish that they had to be abandoned. Obviously this condition was due to subsidence of the slough channels, and to decline of ground-water levels. Well water levels dating from 1902 were obtained for comparison from United States Geological Survey (U.S.G.S.) publications (10)(25)(26) and from the U.S.G.S. office in

[7a] Numerals in parentheses, thus: (24), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

Los Angeles, as well as from the owners themselves. These showed the cause; the level lines, the effect. The lowering of the bench marks was also corroborated by the office of the United States Coast and Geodetic Survey.

This paper, as well as one covering the same area and subject (27) is of value in "pointing up," by direct analogy with the exhaustion of the valuable mineral resource, petroleum, the fact that the even more valuable resource to the semi-arid west—ground water—is being exhausted by the gradual elimination of ground storage capacity through the contraction of the underground void space under the weight of the overburden with the lowering of ground-water levels through overdraft. It has long been a matter of concern to the writer, and has become particularly acute during the severe winter drought of 1947–1948 in California and Nevada. The writer has recommended shutting down the pumping plants of the well field of the City of Palo Alto (Calif.) and purchasing the total domestic supply from San Francisco's Hetch Hetchy surface water supply in order to conserve what ground-water storage capacity remains. He has also opposed the drilling of new wells for clients in the Santa Clara and San Joaquin valleys of California, because there is now more well capacity in the basins concerned than there is ground-water capacity to satisfy it. Overdraft will reduce water levels, allow contraction of pore space, and thus reduce ground-water storage capacity.

In view of the facts, the writer is of the opinion that the accelerated subsidence of the Terminal Island area since 1941 is attributable, in the main, to the dewatering, through continued pumpage necessary for constructing the dry dock, of the sand and other surface deposits down to El. −75. The tests on the shale samples are interesting, yet they are in no way conclusive, nor do they change the writer's previously expressed opinion (28)(29) that the application of the theories of soil mechanics and the results of tests upon small samples are of value only as a check on judgment based on determinable facts and should not be used to evaluate cause and effect in numerical terms.

Bibliography.—

(10) **"Development of Underground Waters in the Central Coastal Plain Region of Southern California,"** by Walter C. Mendenhall, *Water-Supply Paper No. 138*, U.S.G.S., U. S. Govt. Printing Office, Washington, D. C., 1905.

(24) *Transactions*, ASCE, Vol. 111, pp. 346–349.

(25) **"Development of Underground Waters in the Eastern Coastal Plain Region of Southern California,"** by Walter C. Mendenhall, *Water-Supply Paper No. 137*, U.S.G.S., U. S. Govt. Printing Office, Washington, D. C., 1905.

(26) **"Development of Underground Waters in the Western Coastal Plain Region of Southern California,"** by Walter C. Mendenhall, *Water-Supply Paper No. 139*, U.S.G.S., U. S. Govt. Printing Office, Washington, D. C., 1905.

(27) **"Vertical Movement in the Los Angeles Region, 1906–1946,"** by Ernest J. Parkin, *Transactions*, Am. Geophysical Union, February, 1948, p. 17.

(28) *Transactions*, ASCE, Vol. 110, pp. 340–341.

(29) *Ibid.*, Vol. 112, pp. 437–438.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ESTIMATING DATA FOR RESERVOIR GATES

Discussion

By A. E. Niederhoff, and Joseph R. Bowman

A. E. Niederhoff,[14] M. ASCE.—The tabulations and charts given by Mr. Boissonnault have filled in gaps in data on reservoir gates. It is very commendable that the author has taken the time to assemble this information and to correlate it for use by other members of the profession. As stated in the "Summary," the charts are of immense value in making preliminary estimates quickly and consistently. Any engineer who has worked in the planning department of the United States Army Engineers knows how necessary it is to have charts such as these for reference. Exception is taken, however, to the use of the charts in checking a detailed and finished design.

The charts yield only traditional or historic values and do not take into account advances made in design. If the final, detailed design of reservoir gates had to conform in weight to values from the charts, no improvement would have been made over former designs. New materials and new techniques of fabrication are available which reduce the weights of gates and it is entirely conceivable that other improvements will be made in the future. The trend is away from steel toward noncorrosive light metals.

The text of the article is excellent in content but one important phase has been omitted. If the author had included a discussion on the applicability of the various types of gates, his paper would have been a veritable treatise. Under what conditions does one use a slide gate? In addition to personal preference, there are several other reasons why slide gates are used instead of Broome or submerged Tainter gates. Similarly, drum gates are used on the crests of dams under certain conditions which make the cheaper Tainter gate inapplicable. Not only are economic items involved, but also hydraulic characteristics and operational factors. Perhaps the author will cover these very significant items in his concluding discussion as well as a typical example of the use of his charts.

Note.—This paper by Frank L. Boissonnault appeared in September, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1948, by Theodore B. Rights; and March, 1948, by George R. Latham, and D. A. Buzzell.

[14] Specification Writer, U. S. Naval Ordnance Test Station, China Lake, Calif.

One important crest gate on dams that was not mentioned is the vertical-lift gate. The writer published a short article on this subject,[15] during the preparation of which it became more and more apparent that the vertical-lift gate is economical for streams having large discharges where ice conditions are unusual. Curves were developed yielding the weight of the gate and the discharge through a gate opening. By the use of formulas the cost of the gate could be obtained, the discharge through a gate could be computed, and the number of gates composing the spillway could be calculated. Through a typographical error in the text the formula for the cost of a gate gave ridiculous results. Using the author's nomenclature and a construction cost index of 116 for the year 1940, this formula should have been:

$$\text{Cost per gate} = 60 \left(\frac{A^2 B^2}{2{,}000}\right)^{0.87} \quad \text{.................} (7)$$

It would be of interest to know if the author checks this formula for cost of vertical-lift gates for dams.

The author mentions that welded Tainter gate weights are in fair agreement with the dashed line shown in Fig. 4, and that weights derived therefrom agree with curves published by the writer. He qualifies the use of this curve with the statement (under the heading, "Radial Crest Gates"):

> "Not many gates of welded construction have been built, but the calculated weights of recent designs seem to show fairly close agreement with the "Welded Construction" curve (see Fig. 4)."

Since World War II, when speed of production, economy, and safety dictated that welded steel ships and welded armored steel tanks were necessary, the knowledge of the art of welding has increased. Many ship building concerns used welding equipment exclusively and did not have the tools for doing riveted construction. As a result it is believed that in the future Tainter gates will be welded and Mr. Boissonnault's dashed line curve in Fig. 4 will represent weights that are more currently correct.

The writer wishes to reiterate that, in spite of a thorough checking and rechecking of this fine paper, he has found little cause for complaint. The author has had the time and opportunity to do a fine job of assembling and correlating information and is to be congratulated on his work.

JOSEPH R. BOWMAN,[16] Assoc. M. ASCE.—A great deal of painstaking work in assembling data on gates and hoists and in correlating the several factors contributing to the cost of such equipment is reflected in this paper. The graphical estimating methods presented by the author should have considerable practical value to engineers engaged in the study of water storage and power projects. Although the general methods employed are not new,[2] much of the data presented concerns recently designed equipment, thus reflecting modern

[15] "Vertical Lift Gate Selection," by A. E. Niederhoff, *Western Construction News*, February, 1944, pp. 73–75.

[16] Hydr. Engr., Harza Eng. Co., Chicago, Ill.

[2] "Hydro-electric Handbook," by W. P. Creager and Joel D. Justin, John Wiley & Sons, Inc., New York, N. Y., 1927, p. 317.

trends in engineering practice. The writer endorses the use of graphical methods for estimating purposes in preference to more direct methods because the practical limitations may be shown more clearly in graphical form.

The particular types of gates with which the paper deals have been given excellent treatment, but the field of extensively used modern reservoir gates has not been fully represented. The general field of vertical-lift gates, of which the Broome gate is but one type, should receive further consideration; consequently, much of this discussion will be devoted to gates in this field.

High-Pressure Slide Gates.—The author has presented a seemingly reliable method of determining the weight of hydraulically operated, high head sluice gates, but Eq. 1 appears to be rather complex for estimating purposes. The writer notes that the weight curve in Fig. 2 has been drawn through points representing only United States Engineer Department (USED) flood control gates, whereas an additional curve, lying somewhat below the author's curve, can be drawn to represent the Bureau of Reclamation (USBR) general-purpose type of high-pressure slide gate.

It is regretted that methods for determination of weights were not presented for high head gates of the ring follower, ring seal, paradox, coaster, and tractor types—as well as for the butterfly, needle, Howell-Bunger, and other types of high-pressure valves. It is hoped that the author may see fit to include supplementary information of this nature in his closing discussion.

The effects of standardization of design on unit prices for the fabrication of gates are well illustrated in Table 2. Although the comparative values shown in this tabulation were derived from prices for high-pressure slide gates, they may be applied with a reasonable degree of accuracy to other types of gates. Further economies are to be realized from standardization of design in the reduction of design costs and other indirect costs.

Tainter Gates.—The writer sees no justification for changing the direction of a gate weight curve to qualify a single (and possibly doubtful) point, as the author has done in the case of the lower end of the middle curve in Fig. 4.

The curves for Tainter gate weights published by A. E. Niederhoff,[5] M. ASCE, replotted in Fig. 4 to express the weights of welded Tainter gates, appear to be quite reliable; inspection of Mr. Niederhoff's article[5] substantiates Mr. Boissonnault's implied belief that the weights expressed by this curve include the combined weights of gate, guides, and anchorages.

The author implies that no relationship can be shown to exist between Tainter gate weights and hoist capacities. However, it can be shown that the majority of points in Figs. 4 and 5 will lie on common curves corresponding to submergible and nonsubmergible types of Tainter gates, respectively, when hoist capacity is plotted against Eq. 3.

Roller and Drum Crest Gates.—Because roller and drum crest gates are usually designed to satisfy the specific requirements of the respective jobs for which they are selected, their weights should be expected to conform to definite characteristic curves. This observation seems to be borne out by the drum gates in Fig. 8, but does not appear to be substantiated by the roller gate points in Fig. 7. Conformity of the drum gate points to a common character-

[5] "Selection of Tainter Gates," by A. E. Niederhoff, *Western Construction News*, December, 1943, p. 555.

istic curve may be due to the more exacting operational requirements imposed on this type of gate.

Broome Gates.—Figs. 10 to 13, inclusive, for the Broome-type caterpillar gates use parameters considerably more complex than expressed in Eq. 3, but the results obtained through the use of such expressions appear to be more consistent than those obtained by the employment of other methods of analysis. However, the weights of these gates may be indicated approximately in terms of Eq. 3 with sufficient accuracy for preliminary comparison with other types of gates, as will be shown later.

Vertical-Lift Gates in General.—Both the Broome and fixed-wheel types of vertical-lift gates have been used extensively in the low head field as submerged intake gates. Both may be used advantageously as spillway crest gates, particularly where large flows are to be passed over comparatively short regulated spillways; their ability to be raised entirely clear of the water surface gives these gates a further advantage over Tainter gates. Although the Broome gate generally has better seating characteristics than the fixed-wheel gate, the latter has the distinct advantage of considerably lower gate cost. Moreover,

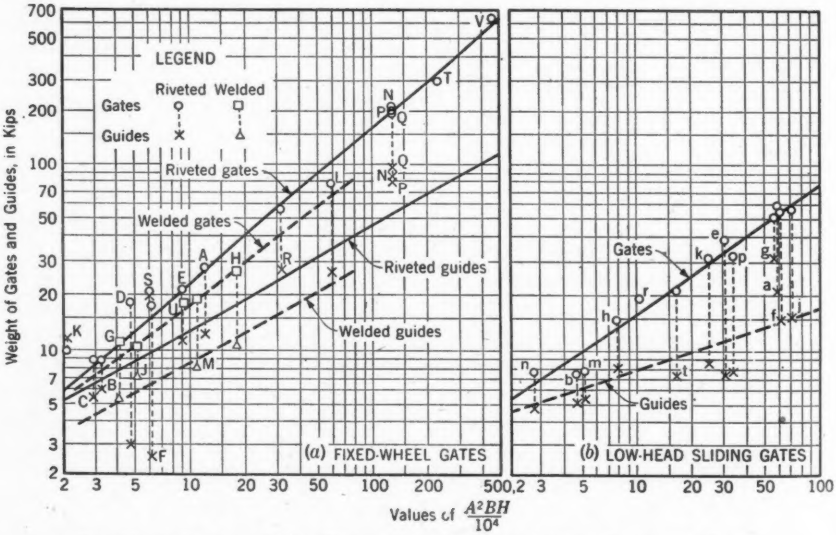

FIG. 15.—WEIGHT OF FIXED-WHEEL AND LOW HEAD SLIDING GATES AND GUIDES

the fixed-wheel gate can be split readily into two or more individually handled sections, thereby materially reducing hoist costs with but a small increase in the weight of the complete gate.

Although Broome and fixed-wheel gates may be operated against either full or balanced heads, low head sliding gates are restricted to operation against balanced heads because of their relatively high frictional resistance, and hence are used primarily as bulkheads. Gates of this type have been used on spillway crests as temporary bulkheads, and at hydro plants to close draft tube outlets so that the water passages may be unwatered. Sliding bulkhead gates are

frequently used in hydro plant intakes in conjunction with Broome or fixed-wheel gates, thus effecting a saving in intake gate costs.

Fixed-Wheel Gates.—The weights of several fixed-wheel gates and their guides (embedded parts) are shown in Fig. 15(*a*); accompanying dimensional and cost data are given in Table 8. The majority of these gates are of a conventional type of construction, wherein the skin plate is welded to a framework of standard structural shapes with riveted connections. The development of all-welded gates has progressed to an extent which permits their segregation from gates employing older methods of construction. The writer has found it advantageous to be able to make a distinction between gate and guide weights; accordingly, the points and weight curves shown in Fig. 15(*a*) indicate the separate weights of gates and guides for both riveted and welded types of construction. Weight curves developed for fixed-wheel spillway crest gates by Mr. Niederhoff,[15] when replotted in terms of Eq. 3, coincide with the writer's weight curve for riveted gates (exclusive of guides)—thus applying to intake as well as spillway crest gates.

TABLE 8.—DATA ON FIXED-WHEEL GATES SHOWN IN FIG. 15(*a*)

Gate	No. of gates	DIMENSIONS OF OPENING			Year of purchase	UNIT COSTS (1939 INDEX) ($)	
		A	*B*	*H*		F.O.B. site	Installed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A[ab]	1	14.25	19.33	30.33	1927	0.140	....
B[ab]	1	11.25	9.00	31.5	1928	0.157	....
C[a]	2	11.7	10.12	21.33	1928	0.065	....
D[ac]	1	18.0	13.5	10.75	1928	0.070	....
E[a]	2	14.25	14.83	29.6	1930	0.096	....
F[ac]	10	35.0	10.0	5.0	1930	0.067	....
G[ad]	3	14.6	13.0	14.5	1931	0.069	....
H[ad]	3	20.0	20.0	22.3	1936	0.088	....
J[ad]	6	13.25	14.5	19.75	1936	0.089	....
K[a]	1	10.2	10.2	19.9	1936	....	....
L[a]	2	15.3	39.0	64.5	1939	0.095	....
M[ad]	4	13.0	16.9	38.0	1948	....	....
N[ce]	18	40.0	40.0	20.0	1938	0.110	0.130
P[ce]	22	40.0	40.0	20.0	1938	0.108	0.129
Q[ce]	18	40.0	40.0	20.0	1939	0.096	0.122
R[e]	1	14.5	22.3	66.9	1942	0.246	0.355
S[e]	1	12.0	12.75	32.6	1942	0.177	0.285
T[ce]	..	40.0	50.0	25.0	....	....	....
U[d]	2	15.5	15.5	25.75	....	....	....
V[e]	..	50.0	60.0	30.0	....	....	....

[a] Installed on projects designed by consulting engineer. [b] Designed by manufacturers. [c] Spillway crest gates; remainder, power intake gates. [d] All-welded construction. [e] Designed by the Tennessee Valley Authority (TVA); corrosion allowance of 1/16 in. added to skin plates and principal frame members.

The points representing gate K lie considerably above their respective weight curves; the gate was designed for a probable future increase in operating head, and the guides presumably were designed to reduce installation costs at a small increase in first cost. Unfortunately, installation cost data to substantiate the latter conclusion are not available to the writer. The guides of gates N, P, and Q consist of rails mounted on steel towers embedded in the concrete piers;[17] the accomplishments of this design were twofold in that accurate field alinement of the guides was assured at low unit costs of installation.

[17] "The Pickwick Landing Project," *Technical Report No. 3*, TVA, Govt. Printing Office, Washington, D.C., 1941, p. 230.

The writer has indicated no graphical relationship between gate weights and weights or capacities of hoists for fixed-wheel gates, as the methods of application of hoisting equipment in the hydroelectric power field vary considerably. However, in the case of spillway crest gates having wheels equipped with antifriction bearings, the capacity of the hoist should at least equal the weight of the gate plus frictional resistance of about 3% of the total water pressure on the gate. In considering the use of a traveling hoist, the weight of a lifting beam or other device should be added; the weight of a lifting beam depends on its application and may vary from 5% to 25% of the weight of the gate. Hoist capacities may be reduced by sectionalizing gates handled by traveling hoists, or by counterweighting gates lifted by fixed hoists. Hoist capacities for power intake gates vary widely, depending on the method

TABLE 9.—SUMMARY OF ECONOMIC STUDY FOR SELECTION OF DESIGN FOR ALL-WELDED GATE M IN FIG. 15(*a*)

Design	Location of:		Construction	Weight (lb)		Required hoist capacity (tons)
	Skin plate	Bottom seal		Gate only	Ballast (seating)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1M.......	Upsteam	Downstream	Standard shapes	19,000	29,000	27.5
2M.......	Upstream	Upstream	Built-up plates	16,000	None	35.0
3M.......	Downstream	Downstream	Built-up plates	16,000	3,000	20.0
4M.......	Downstream	Downstream	Standard shapes	19,000	None	20.0

of operation (whether against full or balanced heads) and on the design of the gates themselves. A typical example of the effect of gate design on hoist capacity is given in Table 9, which summarizes a study made to determine the most economical design for gate M in Fig. 15(*a*); this gate is designed for operation against full hydrostatic head, and its wheels are equipped with grease-lubricated roller bearings. Price estimates furnished by a gate manufacturer

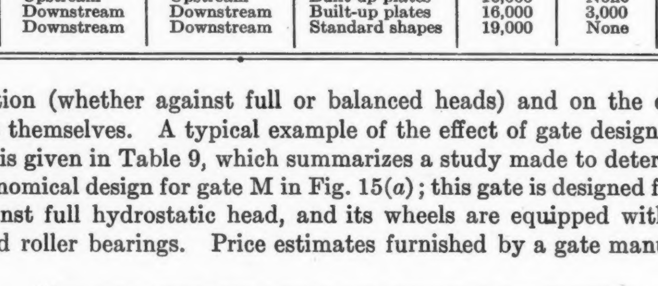

FIG. 16.—WEIGHT OF OUTDOOR GANTRY CRANE GATE HOISTS

indicate that the cost of a 16,000-lb gate of built-up plate construction would be approximately the same as the cost of a 19,000-lb gate of standard shape construction. Thus, the cost of gate 3M is greater than that of gate 4M by the cost of ballast required to seat gate 3M.

Fig. 16 shows approximately the variation of crane weight with hoist capacity (based on project reports for several TVA projects) for outdoor gantry cranes. Installed unit costs (adjusted to 1939 index) for outdoor gantry cranes with trolleys and auxiliary hoists ranged from 20 cents to 30 cents per lb.

Low Head Sliding Gates.—The weights of several low head sliding gates and their guides are shown in Fig. 15(*b*); accompanying dimensional and cost data are given in Table 10. Hoists designed exclusively for handling low head sliding gates operated against balanced heads ordinarily require capacities slightly in excess of the weights of the gates. The excess capacity is equivalent to the weight of a lifting device plus a nominal force required to break the gate away from its sealed position; where sloping guides are used, frictional resistance due to the weight of the gate will be encountered. Gates f and j were sectionalized to reduce hoist costs. Fig. 16 also indicates approximately the variation of crane weight with hoist capacity for outdoor gantry cranes with fixed hoists. Installed unit costs (adjusted to 1939 index) for these cranes ranged from 25 cents to 40 cents per lb.

Miscellaneous Types of Gates.—During the period between World War I and World War II many types of water control gates were developed; the gates treated in the paper and in this discussion represent the surviving types more commonly used in modern practice. Occasionally, however, it is necessary to resort to special gate equipment adaptable to unusual site conditions. A few noteworthy examples of the use of special gates include the utilization of butterfly spillway crest gates at Exchequer Dam[18] (California), cylinder intake and outlet gates at Hoover Dam[19] (Arizona and Nevada), and a floating ring

TABLE 10.—DATA ON LOW HEAD SLIDING GATES SHOWN IN FIG. 15(*b*)

Gate	No. of gates	OPENING DIMENSIONS			Year of purchase	UNIT COSTS (1939 INDEX) ($)	
		A	*B*	*H*		F.O.B. site	Installed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
a[a,b]	4	15.3	39.0	64.5	1939	....	....
b[a]	2	13.5	9.75	25.6	1948	....	....
e[c]	6	18.0	24.5	37.75	1935	0.112	....
f[c]	3	25.0	18.0	53.0	1938	0.075	0.088
g[c]	9	18.66	19.83	80.1	1938	0.088	0.134
h[c]	3	13.33	12.0	36.0	1939	0.097	0.199
j[c]	3	25.54	18.0	62.0	1939	0.075	0.096
k[c]	2	20.5	11.0	52.5	1942	0.104	0.169
m[c]	2	11.0	7.75	54.1	1942	0.142	0.282
n[c]	2	10.75	6.50	27.0	1942	0.142	0.236
p[c]	2	20.5	11.0	71.5	1943	0.122	0.231
r[b]	..	15.0	30.0	15.0	1940	....	....
s[d]	..	19.5	14.0	61.4	1947	....	....
t[d]	2	21.0	14.1	26.0	1947	....	....

[a] Designed by consulting engineer. [b] Power intake gates; remainder, powerhouse draft tube gates. [c] TVA projects. [d] All-welded gates.

[18] "Vertical Butterfly Gates on Exchequer Dam," *Engineering News-Record*, August 26, 1926, p. 344.

[19] "Hydraulic Valves and Gates for Boulder Dam—Part I," by P. A. Kinzie, *Mechanical Engineering*, July, 1934, p. 387.

spillway gate at Owyhee Dam[20] (Oregon). The use of such special types of gates is generally so infrequent that progressive improvements in design and construction make the prediction of their weight and cost a complex problem.

Comparison of Types of Gates.—It is frequently desirable to draw quick comparisons between several types of gates under given dimensional conditions. Accordingly, weight curves for the types of gates so treated in the paper and in this discussion are shown in Fig. 17 as functions of Eq. 3. The weights indicated include the gates and their embedded parts.

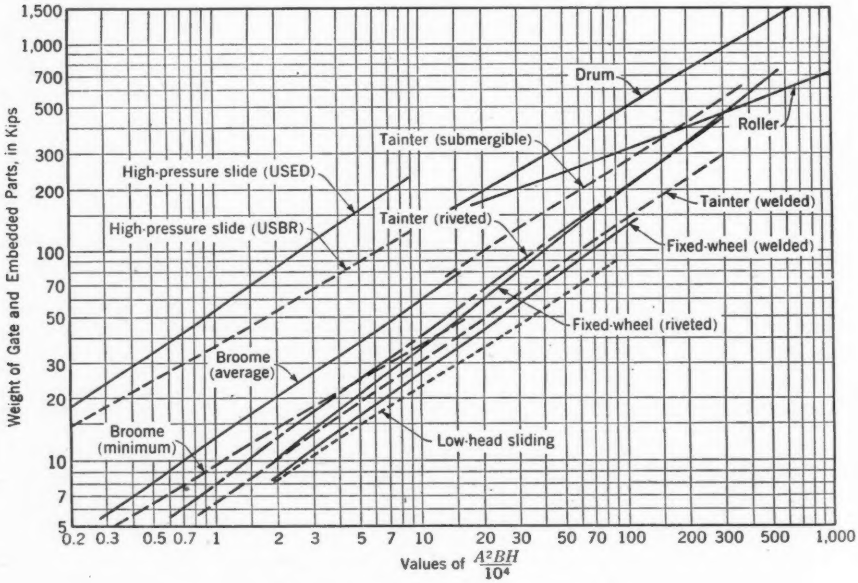

FIG. 17.—COMPARISON OF GATE WEIGHTS

Conclusion.—Mr. Boissonnault has made an important contribution in assembling reliable estimating data for reservoir gates, and it is hoped that the supplementary data given in this discussion will add to the value of the original paper.

Acknowledgment.—The writer should like to express his gratitude to H. G. Gerdes, M. ASCE, for inspiration derived from his unpublished work on gates, and to Kenneth C. Roberts, James S. Bowman, and Calvin V. Davis, Members, ASCE, and E. Montford Fucik, Assoc. M. ASCE, for their many valuable suggestions.

[20] "Floating-Ring Gate and Glory-Hole Spillway on Owyhee Dam," by Lewis G. Smith, *Reclamation Era*, August, 1940, p. 226.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

BEAM DEFLECTIONS BY SECOND AND THIRD MOMENTS

Discussion

BY HSU SHIH-CHANG

HSU SHIH-CHANG,[14] ESQ.—The purpose of this paper was to simplify the procedure for computing beam deflections, thus eliminating the unnecessary computations involved in other methods. For example, the equation for the deflection at any point N of a cantilever beam that supports a single force P

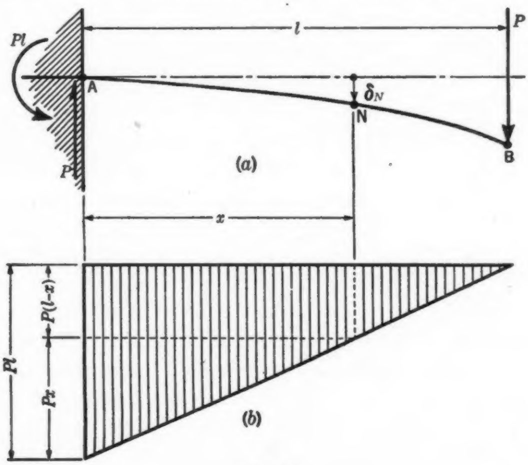

FIG. 23

at the free end (Fig. 23(a)) is determined by area moments in three steps as follows:

(1) Construct the bending moment diagram shown in Fig. 23(b);

(2) Compute the area of the bending moment diagram between points A and N—

$$A_m = -P\,(l - x)\,x - \tfrac{1}{2}\,(P\,x)\,x \ldots\ldots\ldots\ldots\ldots (79)$$

NOTE.—This paper by Hsu Shih-Chang was published in March, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by A. Floris, Robert B. B. Moorman, and Frank J. McCormick; and November, 1947, by William A. Conwell.

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and

(3) Compute the moment of that area about point N—

$$Q_n = -[P(l - x)\,x]\frac{x}{2} - (\tfrac{1}{2}P x^2)\left(\frac{2x}{3}\right) = -\tfrac{1}{6}P x^2 (3l - x) \quad (80)$$

Eq. 80 is to be divided by $E I$ to obtain the equation of deflection:

$$\delta_N = -\frac{P x^2}{6 E I}(3l - x) \quad (81)$$

By the proposed method, it is necessary only to compute the third moment of the reaction P and the couple $P l$ (both at built-in end A) with respect to point N; thus:

$$P x^3 - 3 P l x^2 = -P x^2 (3l - x) \quad (82)$$

Eq. 82 is divided by $6 E I$ to obtain the equation of deflection—

$$\delta_N = -\frac{P x^2}{6 E I}(3l - x) \quad (83)$$

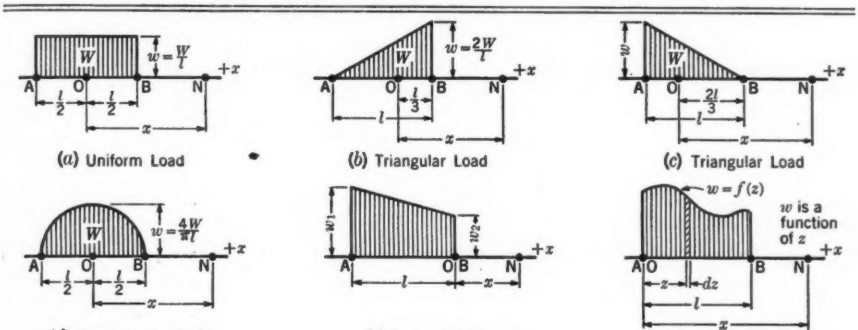
Although the results are the same, the proposed method simplifies the procedure of computation considerably.

Since the moment center of the second and third moments of any force system, either balanced or unbalanced, may be chosen at any point on the reference line according to the characteristics of second and third moments, the computer can select some convenient points as moment centers to make the calculation simple and easy for any given case. Fig. 28, described subsequently, is an illustration of this procedure. Also, using the proposed method, the designer can formulate the general solutions of angle changes, deflections, and statically indeterminate problems, for a given support condition of beam carrying any loading, expressed as functions of the second and third moments of forces and couples. For any given loading condition, the desired results are obtained quickly from the formulas, which are convenient for practical use. Tables 4 and 5 (explained in detail subsequently) are prepared for this purpose.

Since the bending moment on which the other methods depend is very useful in other fields of structural engineering and is more generally employed, and since the concept of second and third moments, as adapted to the problem of this paper, is new, it is natural that it would be difficult for it to replace other methods immediately. As the concept of the second and third moments becomes more familiar, however, and as its usefulness becomes recognized in other fields of engineering (such as the analysis of hyperstatic structures and continuous frames, proposed by Mr. Floris and Professor Moorman), it is possible that it will be found convenient.

The summary of moment formulas in Table 2, proposed by Professor Moorman, is of much practical value. For a force system symmetrical about point O (referring to Fig. 5), it is obvious that the first moment with respect to point O is also equal to zero; and, if the entire force system is divided into two halves at point O, the second moments of each half system with respect to point O must be the same, so that the second moment of the entire system with respect

TABLE 3.—SECOND AND THIRD MOMENTS AT LOADINGS
(Expressed in Terms of Total Load W and Load Intensity w)



(a) Uniform Load (b) Triangular Load (c) Triangular Load (d) Semi-circular Load (e) Trapezoidal Load (f) General Load

SECOND MOMENT			THIRD MOMENT		
Notation	Total load,[a] W	Intensity of load, w	Notation	Total load, W	Intensity of load, w
		(a) UNIFORM LOAD			
M''_A	$-\frac{W\,l^2}{3}$	$-\frac{w\,l^3}{3}$	M'''_A	$+\frac{W\,l^3}{4}$	$+\frac{w\,l^4}{4}$
M''_O	$-\frac{W\,l^2}{12}$	$-\frac{w\,l^3}{12}$	M'''_O	0	0
M''_B	$-\frac{W\,l^2}{3}$	$-\frac{w\,l^3}{3}$	M'''_B	$-\frac{W\,l^3}{4}$	$-\frac{w\,l^4}{4}$
M''_N	$-\frac{W\,l^2}{12} - W\,x^2$	$-\frac{w\,l^3}{12} - w\,l\,x^2$	M'''_N	$-\frac{W\,l^2}{4}\,x - W\,x^3$	$-\frac{w\,l^3}{4}\,x - w\,l\,x^3$
		(b) TRIANGULAR LOAD			
M''_A	$-\frac{W\,l^2}{2}$	$-\frac{w\,l^3}{4}$	M'''_A	$+\frac{2\,W\,l^3}{5}$	$+\frac{w\,l^4}{5}$
M''_O	$-\frac{W\,l^2}{18}$	$-\frac{w\,l^3}{36}$	M'''_O	$-\frac{W\,l^3}{135}$	$-\frac{w\,l^4}{270}$
M''_B	$-\frac{W\,l^2}{6}$	$-\frac{w\,l^3}{12}$	M'''_B	$-\frac{W\,l^3}{10}$	$-\frac{w\,l^4}{20}$
M''_N	$-\frac{W\,l^2}{18} - W\,x^2$	$-\frac{w\,l^3}{36} - \frac{w\,l}{2}\,x^2$	M'''_N	$-\frac{W\,l^3}{135} - \frac{W\,l^2}{6}\,x - W\,x^3$	$-\frac{w\,l^4}{270} - \frac{w\,l^3}{12}\,x - \frac{w\,l}{2}\,x^3$
		(c) TRIANGULAR LOAD			
M''_A	$-\frac{W\,l^2}{6}$	$-\frac{w\,l^3}{12}$	M'''_A	$+\frac{W\,l^3}{10}$	$+\frac{w\,l^4}{20}$
M''_O	$-\frac{W\,l^2}{18}$	$-\frac{w\,l^3}{36}$	M'''_O	$+\frac{W\,l^3}{135}$	$+\frac{w\,l^4}{270}$
M''_B	$-\frac{W\,l^2}{2}$	$-\frac{w\,l^3}{4}$	M'''_B	$-\frac{2\,W\,l^3}{5}$	$-\frac{w\,l^4}{5}$
M''_N	$-\frac{W\,l^2}{18} - W\,x^2$	$-\frac{w\,l^3}{36} - \frac{w\,l}{2}\,x^2$	M'''_N	$+\frac{W\,l^3}{135} - \frac{W\,l^2}{6}\,x - W\,x^3$	$+\frac{w\,l^4}{270} - \frac{w\,l^3}{12}\,x - \frac{w\,l}{2}\,x^3$

TABLE 3.—(*Continued*)

Second Moment			Third Moment		
Notation	Total load,[a] W	Intensity of load, w	Notation	Total load, W	Intensity of load, w
		(*d*) Semicircular Load			
M''_A	$-\frac{5\,W\,l^2}{16}$	$-\frac{5\,w\,\pi\,l^3}{64}$	M'''_A	$+\frac{7\,W\,l^3}{32}$	$+\frac{7\,w\,\pi\,l^4}{128}$
M''_O	$-\frac{W\,l^2}{16}$	$-\frac{w\,\pi\,l^3}{64}$	M'''_O	0	0
M''_B	$-\frac{5\,W\,l^2}{16}$	$-\frac{5\,w\,\pi\,l^3}{64}$	M'''_B	$-\frac{7\,W\,l^3}{32}$	$-\frac{7\,w\,\pi\,l^4}{128}$
M''_N	$-\frac{W\,l^2}{16} - W\,x^2$	$-\frac{w\,\pi\,l^3}{64} - \frac{w\,\pi\,l}{4}\,x^2$	M'''_N	$-\frac{3\,W\,l^2}{16}\,x - W\,x^3$	$-\frac{3\,w\,\pi\,l^3}{64}\,x - \frac{w\,\pi\,l}{4}\,x^3$
		(*e*) Trapezoidal Load			
M''_A		$-\frac{(w_1 + 3\,w_2)\,l^3}{12}$	M'''_A		$+\frac{(w_1 + 4\,w_2)\,l^4}{20}$
M''_B		$-\frac{(3\,w_1 + w_2)\,l^3}{12}$	M'''_B		$-\frac{(4\,w_1 + w_2)\,l^4}{20}$
M''_N		$-\frac{(3\,w_1 + w_2)\,l^3}{12} - \frac{(2\,w_1 + w_2)\,l^2}{3}\,x - \frac{(w_1 + w_2)\,l}{2}\,x^2$	M'''_N		$-\frac{(4\,w_1 + w_2)\,l^4}{20} - \frac{(3\,w_1 + w_2)\,l^3}{4}\,x - \frac{(2\,w_1 + w_2)\,l^2}{2}\,x^2 - \frac{(w_1 + w_2)\,l}{2}\,x^3$
		(*f*) General Load			
M''_A		$-\int_0^l f(z)\,z^2\,dz$	M'''_A		$+\int_0^l f(z)\,z^3\,dz$
M''_B		$-\int_0^l f(z)\,(l - z)^2\,dz$	M'''_B		$-\int_0^l f(z)\,(l - z)^3\,dz$
M''_N		$-\int_0^l f(z)\,(x - z)^2\,dz$	M'''_N		$-\int_0^l f(z)\,(x - z)^3\,dz$

[a] Formulas for total load W are omitted in Tables 3(*e*) and 3(*f*).

to point O is equal to twice the second moment of the half force system with respect to point O. Also, for a balanced force system, the first moment with respect to any point is equal to zero. Therefore, two formulas $M = 0$ and $M_O = 0$ inserted in the third and last lines, respectively, under "first moment" in Table 2 and a formula $M''_O = 2$ (M''_O of half system) inserted in the last line under "second moment" would make that table more complete.

The writer appreciates the suggestion by Professor Moorman that the information of second and third moments of loadings be summarized, and has prepared Table 3. If Eq. 54, theorem II, and Table 3 are applied to the example in Fig. 22, the solution can be simplified as follows: Referring to Fig. 24,

the deflection at point A may be expressed as

$$\begin{aligned}
\delta_A = y_{ZA} &= \Delta\left(\frac{1}{6\,E\,I}\right)_B M'''_{AB,Z} + \frac{1}{6\,E\,I_0} M'''_{AZ,Z} \\
&= \left(\frac{1}{6\,E\,I_1} - \frac{1}{6\,E\,I_0}\right)\left(M'''_{AB,B} + 3\,M''_{AB}\frac{l}{4}\right) \\
&\quad + \frac{1}{6\,E\,I_0} M'''_{AZ,Z} \\
&= \left(\frac{1}{6\,E\,I_1} - \frac{1}{6\,E\,I_0}\right)\left\{3\left(\frac{w\,l^2}{8}\right)\left(\frac{l}{4}\right)^2 - \tfrac{1}{4}\,w\left(\frac{l}{4}\right)^4\right. \\
&\quad \left. + 3\left[2\left(\frac{w\,l^2}{8}\right)\left(\frac{l}{4}\right) - \tfrac{1}{3}\,w\left(\frac{l}{4}\right)^3\right]\left(\frac{l}{4}\right)\right\} \\
&\quad + \frac{1}{6\,E\,I_0}\left[3\left(\frac{w\,l^2}{8}\right)\left(\frac{l}{2}\right)^2 - \tfrac{1}{4}\,w\left(\frac{l}{2}\right)^4\right] \\
&= \frac{w\,l^4}{6(32)^2\,E}\left(\frac{67}{I_1} + \frac{13}{I_0}\right) \qquad \text{(84)}
\end{aligned}$$

The advantage of applying Eq. 54 may be further shown by another numerical example. Let it be required to determine the equation for the deflec-

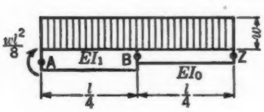


FIG. 24

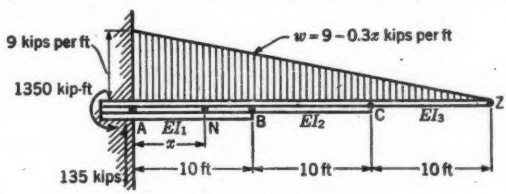


FIG. 25

tion at any point for the beam shown in Fig. 25. Applying Table 3(*e*) and computing the second and third moments,

$$\begin{aligned}
M'''_{AN,N} &= 135\,x^3 - 3 \times 1{,}350\,x^2 - \frac{(4 \times 9 + 9 - 0.3\,x)\,x^4}{20} \\
&= x^2\,(-\,4{,}050 + 135\,x - 2.25\,x^2 + 0.015\,x^3) \qquad \text{(85)}
\end{aligned}$$

Substituting $x = 10$ and $x = 20$, respectively, into Eq. 85, $M'''_{AB,B} = -\,291{,}000$; and $M'''_{AC,C} = -\,852{,}000$; also

$$M''_{AB} = 135 \times 10^2 - 2 \times 1{,}350 \times 10 - \frac{(3 \times 9 + 9 - 0.3 \times 10)\,10^3}{12} = -\,16{,}250$$

and

$$M''_{AC} = 135 \times 20^2 - 2 \times 1{,}350 \times 20 - \frac{(3 \times 9 + 9 - 0.3 \times 20)\,20^3}{12} = -\,20{,}000$$

Then, applying Eq. 54, the deflection at any point N is: For $0 \leqq x \leqq 10$ ft—

$$\delta_N = \frac{1}{6\,E\,I_1} M'''_{AN,N} = \frac{x^2}{6\,E\,I_1}\,(-\,4{,}050 + 135\,x - 2.25\,x^2 + 0.015\,x^3) \qquad \text{(86)}$$

and, for 10 ft $\leqq x \leqq$ 20 ft—

$$\delta_N = \Delta \left(\frac{1}{6 E I}\right)_B M'''_{AB,N} + \frac{1}{6 E I_2} M'''_{AN,N}$$

$$= \left(\frac{1}{6 E I_1} - \frac{1}{6 E I_2}\right) [M'''_{AB,B} + 3 M''_{AB} (x - 10)]$$

$$+ \frac{1}{6 E I_2} M'''_{AN,N}$$

$$= \left(\frac{1}{6 E I_1} - \frac{1}{6 E I_2}\right) [- 291{,}000 + 3 (- 16{,}250) (x - 10)]$$

$$+ \frac{x^2}{6 E I_2} (- 4{,}050 + 135 x - 2.25 x^2 + 0.015 x^3)$$

$$= \frac{125}{E I_1} (262 - 65 x) + \frac{1}{6 E I_2} (- 196{,}500 + 48{,}750 x - 4{,}050 x^2$$

$$+ 135 x^3 - 2.25 x^4 + 0.015 x^5) \qquad (87)$$

Similarly, for 20 ft $\leqq x \leqq$ 30 ft:

$$\delta_N = \Delta \left(\frac{1}{6 E I}\right)_B M'''_{AB,N} + \Delta \left(\frac{1}{6 E I}\right)_C M'''_{AC,N} + \frac{1}{6 E I_3} M'''_{AN,N}$$

$$= \left(\frac{1}{6 E I_1} - \frac{1}{6 E I_2}\right) [- 291{,}000 + 3 (- 16{,}250) (x - 10)]$$

$$+ \left(\frac{1}{6 E I_2} - \frac{1}{6 E I_3}\right) [- 852{,}000 + 3 (- 20{,}000) (x - 20)]$$

$$+ \frac{x^2}{6 E I_3} (- 4{,}050 + 135 x - 2.25 x^2 + 0.015 x^3)$$

$$= \frac{125}{E I_1} (262 - 65 x) + \frac{125}{E I_2} (202 - 15 x)$$

$$+ \frac{1}{6 E I_3} (- 348{,}000 + 60{,}000 x - 4{,}050 x^2$$

$$+ 135 x^3 - 2.25 x^4 + 0.015 x^5) \qquad (88)$$

For the arrangement shown in Fig. 20(*a*), the beam may be cut into two parts at the hip, each part being considered as a straight beam, and solved separately by the proposed method. The writer cannot agree with Professor Moorman's proposal[14a] to modify point O of theorem III. By statics, the resultant of an unbalanced force system can be represented by a single force and a couple both acting at any point on the reference line. Therefore, point O of theorem III is an independent point on the reference line similar to that of theorem II, and can be selected at will. In applying theorem III, point O is chosen before computing the resultant at point O.

Mr. Floris suggests that the second moments of loadings are equivalent to the moments of inertia; thus the values of second moments of various shapes of loadings can be obtained from the values of moments of inertia given in

[14a] Correction for *Transactions:* Change "(O)" to "O" in line 8, page 295, March, 1947, *Proceedings*, and in the next to the last line on page 1159 of September, 1947, *Proceedings*.

reference books. Unfortunately there is no characteristic in the moment of inertia that corresponds to the second moment of a couple, so the analogy of moment of inertia to second moment is only partial. Although the values of second moments of loadings can be taken from reference books by the analogy of moment of inertia to second moment, it is difficult to find a similar analogy to the more important term—third moments. Therefore, it is as well to obtain values of both second and third moments from a separate table, such as Table 3.

The second and third moments of a balanced force system with respect to any point on the reference line have some geometrical significance. Fig. 26(*a*)

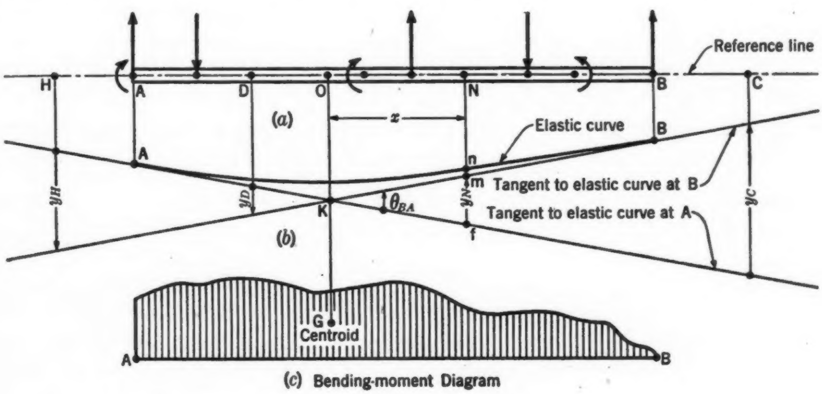


FIG. 26

shows a balanced force system acting on the straight beam AB with the undeflected position of the beam axis as the reference line. Fig. 26(*b*) shows the elastic curve of beam AB, where the two tangents to the elastic curve at ends A and B intersect at point K. Applying theorems I and VI,

$$M''_{AB} = 2\,E\,I\,\theta_{BA} = \text{a constant} \ldots\ldots\ldots\ldots\ldots (89)$$

Take any point N on the reference line, and let y_N be the intercept of the vertical line through point N between the two tangents to the elastic curve at points A and B, as shown in Figs. 26(*a*) and 26(*b*). Ordinate y_N will be positive when measured upward from the tangent at point A to the tangent at point B. Applying theorem VII, Eq. 31*a* gives

$$y_N = \overline{\text{fm}} = \overline{\text{fn}} - \overline{\text{mn}} = y_{NA} - y_{NB} = \frac{M'''_{AN,N}}{6\,E\,I} - \left(-\frac{M'''_{NB,N}}{6\,E\,I}\right)$$
$$= \frac{1}{6\,E\,I}\left(M'''_{AN,N} + M'''_{NB,N}\right) = \frac{1}{6\,E\,I}\,M'''_{AB,N} \ldots (90)$$

or

$$M'''_{AB,N} = 6\,E\,I\,y_N \ldots\ldots\ldots\ldots\ldots\ldots\ldots (91)$$

Similarly, $M'''_{AB,C} = 6\,E\,I\,y_C$; $M'''_{AB,D} = 6\,E\,I\,y_D$; and $M'''_{AB,H} = 6\,E\,I\,y_H$ —in which the values of $M'''_{AB,D}$ and $M'''_{AB,H}$ must be negative, because y_D and y_H are measured downward. Since $y_O = 0$ at the intersection K, it is

obvious that

$$M'''_{AB,O} = 0 \quad \text{.......................} \quad (92)$$

Eqs. 89, 91, and 92 define the geometrical significance of the second and third moments of a balanced force system, acting on a straight beam AB (point B being on the right side of point A), as follows:

(1) The second moment, with respect to any point on the reference line, is equal to the angle between the two tangents to the elastic curve at points A and B, measured from the tangent at point A to the tangent at point B, multiplied by 2 $E I$. Therefore, the second moment is a constant.

(2) The third moment, with respect to any point N on the reference line, is equal to the intercept of the vertical line through point N between the two tangents at points A and B, measured from the tangent at point A to the tangent at point B, multiplied by 6 $E I$.

(3) The third moment, with respect to point O, which is on the same vertical line as the intersection K of the two tangents at points A and B, is equal to zero.

Comparing the elastic curve, Fig. 26(*b*), with the corresponding bending moment diagram, Fig. 26(*c*), since the intersection K and the centroid G of the area of the bending moment diagram occupy the same vertical line,[15] it is interesting to note that the three points—point O which is taken as the moment center of the balanced force system to obtain zero third moment, the intersection K of the two tangents to the elastic curve at points A and B, and the centroid G of the area of the bending moment diagram—are all on the same vertical line. To determine the position of point O, point K, or point G requires the application of theorem II, Eq. 12; thus:

$$M'''_{AB,N} = M'''_{AB,O} + 3\,M''_{AB}\,x \quad \text{...............} \quad (93)$$

in which x (equal to the distance ON) is positive when point M is on the right side of point O. From Eq. 92, since $M'''_{AB,O} = 0$, Eq. 93 reduces to

$$M'''_{AB,N} = 3\,M''_{AB}\,x \quad \text{..................} \quad (94a)$$

or

$$x = \frac{M'''_{AB,N}}{3\,M''_{AB}} \quad \text{......................} \quad (94b)$$

Eq. 94*b* gives the distance from any point N to point O. The position of point O, once determined, may be selected as the origin. Then, from Eq. 94*a*, the third moment of the balanced force system, with respect to any point N on the reference line, can be computed easily as the distance x multiplied by a constant 3 M''_{AB}.

The derivation of Eqs. 52 can be simplified, as follows: Referring to Fig. 27 (which is similar to Fig. 13), Eq. 91 yields

$$y_{ZA} = \overline{aZ} = \overline{ab} + \overline{bc} + \overline{cZ} = \frac{1}{6\,(E\,I)_{AB}}\,M'''_{AB,Z} + \frac{1}{6\,(E\,I)_{BC}}\,M'''_{BC,Z} + \frac{1}{6\,(E\,I)_{CZ}}\,M'''_{CZ,Z} \quad \text{..} \quad (95a)$$

[15] "Improved Method of Finding Beam Deflections," by Ralph W. Stewart, *Civil Engineering*, February, 1934, p. 88, Fig. 1.

Therefore, the general formula is

$$y_{ZA} = \sum \frac{1}{6\,(E\,I)_{AB}}\, M'''_{AB,\,Z} \qquad \text{(95b)}$$

For the beam with both ends built in and on the same level (in which case the two tangents to the elastic curve at the ends of the beam coincide with each other), Eqs. 89 and 91 make evident the fact that the first moment, second moment, and third moment of the balanced force system acting on the beam, with respect to any point on the reference line, are all equal to zero. Since

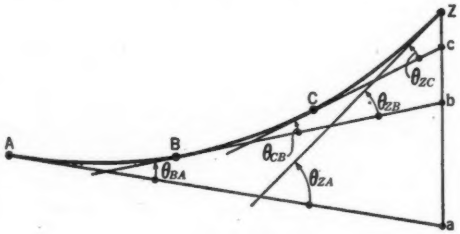


FIG. 27

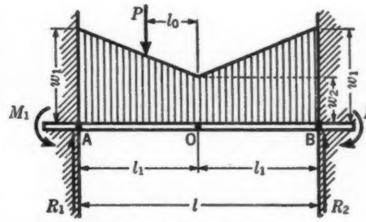


FIG. 28

the moment center of the first, second, and third moments may be taken at any point on the reference line, it is a matter of technique to choose the convenient moment center on the reference line for a given case in order to make the computation as simple and easy as possible. For example, let it be required to determine the two moments M_1 and M_2 at the ends of the beam shown in Fig. 28. The four equations for solving four unknowns M_1, M_2, R_1, and R_2 are

$$R_1 + R_2 = P + l_1\,(w_1 + w_2) \qquad \text{(96a)}$$

$$M_{AB} = 0 \qquad \text{(96b)}$$

$$M''_{AB} = 0 \qquad \text{(96c)}$$

and

$$M'''_{AB} = 0 \qquad \text{(96d)}$$

in which the subscripts following the subscripts AB in symbols M_{AB}, M''_{AB}, and M'''_{AB} are all unnecessary since the location of the moment center is immaterial. From inspection of Fig. 28, the two trapezoidal loads are symmetrical about point O. Therefore, the first and third moments of the two trapezoidal loads with respect to point O are both equal to zero, and the second moment of the two trapezoidal loads with respect to point O is equal to twice the second moment of one trapezoidal load only with respect to point O. Also, point O is in the center of beam AB. Therefore, the most convenient point of moment center for determining M_1 and M_2 is point O. By Eqs. 96*b* and 96*d*,

$$M_{AB,\,O} = R_1\,l_1 - M_1 - P\,l_0 - R_2\,l_1 + M_2 = 0 \qquad \text{(97a)}$$

and

$$M'''_{AB,\,O} = R_1\,l^3_1 - 3\,M_1\,l^2_1 - P\,l^3_0 - R_2\,l^3_1 + 3\,M_2\,l^2_1 = 0 \qquad \text{(97b)}$$

Eliminating the two unknowns R_1 and R_2, both of which occur in Eqs. 97,

$$M_1 - M_2 = \frac{P\, l_0\, (l^2_1 - l^2_0)}{2\, l^2_1} \qquad (98)$$

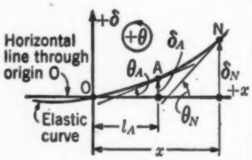
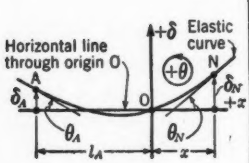
From Eq. 96*c* and Table 3(*e*),

$$M''_{AB,O} = (R_1 + R_2)\, l^2_1 - 2\, M_1\, l_1 - 2\, M_2\, l_1 - P\, l^2_0 - 2 \times \frac{(3\, w_1 + w_2)\, l^3_1}{12} = 0 \qquad (99)$$

Substituting the value of $(R_1 + R_2)$ from Eq. 96*a* into Eq. 99 and simplifying,

$$M_1 + M_2 = \frac{6\, P\, (l^2_1 - l^2_0) + l^3_1\, (3\, w_1 + 5\, w_2)}{12\, l_1} \qquad (100)$$

TABLE 4.—FORMULAS OF ANGLE CHANGES AND DEFLECTIONS[a]

Diagram	Boundary condition	Angle change θ_N of any point N	Deflection δ_N of any point N
(*a*) POINT A ON RIGHT SIDE OF ORIGIN O			
	θ_A (given)	$\frac{1}{2\,E\,I}(-M''_{OA} \pm M''_{ON}) + \theta_A$	$\frac{1}{6\,E\,I}(-3\,M''_{OA}\,x \pm M'''_{ON,N}) + \theta_A\,x$
	$\theta_A = 0$	$\frac{1}{2\,E\,I}(-M''_{OA} \pm M''_{ON})$	$\frac{1}{6\,E\,I}(-3\,M''_{OA}\,x \pm M'''_{ON,N})$
	δ_A (given)	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A} \pm 3\,l_A\,M''_{ON}) + \frac{\delta_A}{l_A}$	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A}\,x \pm l_A\,M'''_{ON,N}) + \frac{\delta_A}{l_A}\,x$
	$\delta_A = 0$	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A} \pm 3\,l_A\,M''_{ON})$	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A}\,x \pm l_A\,M'''_{ON,N})$
(*b*) POINT A ON LEFT SIDE OF ORIGIN O			
	θ_A (given)	$\frac{1}{2\,E\,I}(M''_{OA} \pm M''_{ON}) + \theta_A$	$\frac{1}{6\,E\,I}(3\,M''_{OA}\,x \pm M'''_{ON,N}) + \theta_A\,x$
	$\theta_A = 0$	$\frac{1}{2\,E\,I}(M''_{OA} \pm M''_{ON})$	$\frac{1}{6\,E\,I}(3\,M''_{OA}\,x \pm M'''_{ON,N})$
	δ_A (given)	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A} \pm 3\,l_A\,M''_{ON}) - \frac{\delta_A}{l_A}$	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A}\,x \pm l_A\,M'''_{ON,N}) - \frac{\delta_A}{l_A}\,x$
	$\delta_A = 0$	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A} \pm 3\,l_A\,M''_{ON})$	$\frac{1}{6\,E\,I\,l_A}(-M'''_{OA,A}\,x \pm l_A\,M'''_{ON,N})$

[a] The values of x and the "±" signs before M''_{ON} and $M'''_{ON,N}$ all take positive sign when point N is on the right side of origin O.

Solving Eqs. 98 and 100 simultaneously,

$$M_1 = \frac{1}{24\, l^2_1} [6\, P\, (l_1 + l_0)^2\, (l_1 - l_0) + l^4_1\, (3\, w_1 + 5\, w_2)] \ldots . (101a)$$

and

$$M_2 = \frac{1}{24\, l^2_1} [6\, P\, (l_1 + l_0)\, (l_1 - l_0)^2 + l^4_1\, (3\, w_1 + 5\, w_2)] \ldots . (101b)$$

Table 4 gives the formulas for angle changes and deflections in a beam carrying any loading under various given conditions. In applying this table,

TABLE 5.—GENERAL SOLUTIONS OF STATICALLY INDETERMINATE PROBLEMS
(Assume the Supports to Be Unyielding)

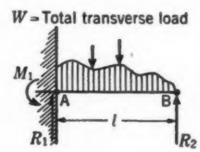
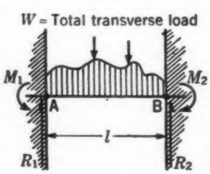
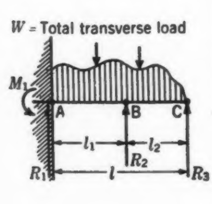
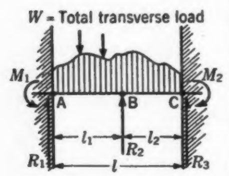
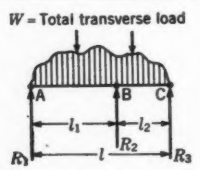
Support condition	REACTIONS AND STATICALLY INDETERMINATE COUPLES	
	Unknown	Solution
(a) BEAM WITH ONE FIXED END AND THE OTHER END FREELY SUPPORTED		
	M_1....	$\frac{1}{2\, l^2} (M'''\overline{AB}, B - l^2\, M\overline{AB}, B)$
	R_1.....	$\frac{1}{2\, l^3} (M'''\overline{AB}, B - 3\, l^2\, M\overline{AB}, B)$
	R_2.....	$-\frac{1}{2\, l^3} (M'''\overline{AB}, B - 3\, l^2\, M\overline{AB}, B) + W$
(b) BEAM WITH BOTH ENDS FIXED		
	M_1....	$\frac{1}{l^2} (M'''\overline{AB}, B - l\, M''\overline{AB}, B)$
	R_1.....	$\frac{1}{l^3} (2\, M'''\overline{AB}, B - 3\, l\, M''\overline{AB}, B)$
	M_2....	$-\frac{1}{l^2} (M'''\overline{AB}, B - 2\, l\, M''\overline{AB}, B) - M\overline{AB}, B$
	R_2.....	$-\frac{1}{l^3} (2\, M'''\overline{AB}, B - 3\, l\, M''\overline{AB}, B) + W$
(c) BEAM ON THREE SUPPORTS WITH ONE END FIXED		
	M_1....	$\frac{1}{l^2_1\, l_2\, (3\, l + l_2)} [l\, (l + l_2)\, M'''\overline{AB}, B - l^2_1\, M'''\overline{AC}, C + l^2_1\, l^2_2\, M\overline{AC}, C]$
	R_1.....	$\frac{1}{l^3_1\, l_2\, (3\, l + l_2)} [(3\, l^2 - l^2_2)\, M'''\overline{AB}, B - 3\, l^2_1\, M'''\overline{AC}, C + 3\, l^2_1\, l^2_2\, M\overline{AC}, C]$
	R_2.....	$-\frac{1}{l^3_1\, l^2_2\, (3\, l + l_2)} [2\, l^3\, M'''\overline{AB}, B - l^2_1\, (2\, l + l_2)\, M'''\overline{AC}, C + 3\, l^2_1\, l_2\, l^2\, M\overline{AC}, C]$
	R_3.....	$\frac{1}{l_1\, l^2_2\, (3\, l + l_2)} [(2\, l + l_2)\, M'''\overline{AB}, B - 2\, l_1\, M'''\overline{AC}, C + 3\, l_1\, l_2\, (l + l_2)\, M\overline{AC}, C] + W$

TABLE 5.—(*Continued*)

Support condition	Reactions and Statically Indeterminate Couples: Unknown	Solution
(*d*) Beam on Three Supports with Both Ends Fixed		
	M_1....	$\frac{1}{2\,l_1^2\,l_2\,l}\left(l^2\,M'''_{\overline{AB},B} - l_1^2\,M'''_{\overline{AC},C} + l_1^2\,l_2\,M''_{\overline{AC},C}\right)$
	R_1.....	$\frac{1}{2\,l_1^3\,l_2\,l}\left[l\,(l + 2\,l_1)\,M'''_{\overline{AB},B} - 3\,l_1^2\,M'''_{\overline{AC},C} + 3\,l_1^2\,l_2\,M''_{\overline{AC},C}\right]$
	R_2.....	$-\frac{1}{2\,l_1^3\,l_2^3}\left[l^3\,M'''_{\overline{AB},B} - l_1^2\,(l + 2\,l_2)\,M'''_{\overline{AC},C} + 3\,l_1^2\,l_2\,l\,M''_{\overline{AC},C}\right]$
	M_2....	$\frac{1}{2\,l_1\,l_2^2\,l}\left[l^2\,M'''_{\overline{AB},B} - l_1\,(l + l_2)\,M'''_{\overline{AC},C} + l_1\,l_2\,(3\,l + l_2)\,M''_{\overline{AC},C}\right] - M_{\overline{AC},C}$
	R_3.....	$\frac{1}{2\,l_1\,l_2^3\,l}\left[l\,(l + 2\,l_2)\,M'''_{\overline{AB},B} - l_1\,(l + 3\,l_2)\,M'''_{\overline{AC},C} + 3\,l_1\,l_2\,(l + l_2)\,M''_{\overline{AC},C}\right] + W$
(*e*) Beam on Three Supports		
	R_1.....	$\frac{1}{2\,l_1^2\,l_2\,l}\left(l\,M'''_{\overline{AB},B} - l_1\,M'''_{\overline{AC},C} + l_1\,l_2^2\,M_{\overline{AC},C}\right)$
	R_2.....	$-\frac{1}{2\,l_1^2\,l_2^2}\left[l\,M'''_{\overline{AB},B} - l_1\,M'''_{\overline{AC},C} + l_1\,l_2\,(l + l_1)\,M_{\overline{AC},C}\right]$
	R_3.....	$\frac{1}{2\,l_1\,l_2^2\,l}\left[l\,M'''_{\overline{AB},B} - l_1\,M'''_{\overline{AC},C} + l_1\,l_2\,(2\,l + l_2)\,M_{\overline{AC},C}\right] + W$

the origin O can be selected at any point on the elastic curve. The angle change or deflection at point A is given by problem or may be found from the support condition of beam. The angle change and deflection at any point are obtained directly from the formulas. All the formulas in Table 4 can be derived by Eqs. 29 and 31. Since the derivations are somewhat complicated, they are omitted from this discussion.

Table 5 gives the general solutions of statically indeterminate problems for some customary support conditions. The beam must be straight and must possess a constant flexural rigidity. The supports are assumed to be unyielding. The loading on the beam may be any transverse loads and any couples (each of the couples acting at a point on the beam axis, like that shown in Fig. 6(*b*)), where the couples are not shown in the table. Symbol W denotes the total transverse load, the reactions at supports being excluded; and $M_{\overline{AB},B}$, $M''_{\overline{AB},B}$, and $M'''_{\overline{AB},B}$ represent the first, second, and third moments, respectively, with respect to point B, of the external loading only, on span AB—end shears and end moments being excluded. The reactions at supports are assumed positive when they act upward. The statically indeterminate couples are assumed positive when their directions of rotation are as indicated in the

table, respectively. The solutions in each case of Table 5 are obtained by solving simultaneous equations. Their derivations are cumbersome and lengthy, and are omitted.

Tables 4 and 5 are advantageous for practical use. For example, let it be required to determine the deflection at the free end D for the beam shown in Fig. 29(*a*). Cut the beam at point C into two parts as shown in Figs. 29(*b*) and 29(*c*). Consider the end shear of 2 kips and the end moment of 8 kip-ft

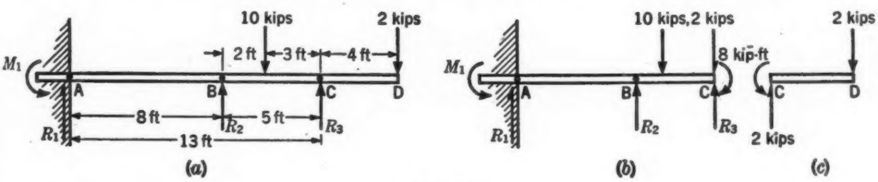


FIG. 29

at end C of beam ABC in Fig. 29(*b*) as the external force and couple, respectively. From the solution for R_3 in Table 5(*c*), substitute 8 ft for l_1, 5 ft for l_2, 13 ft for l, 12 kips for W, and zero for $M'''_{\overline{AB},B}$; thus:

$$R_3 = 12 + \frac{1}{8 \times 5^2 \times (3 \times 13 + 5)} [- 2 \times 8\, M'''_{\overline{AC},C} + 3 \times 8 \times 5\,(13 + 5)\, M_{\overline{AC},C}]$$

$$R_3 = 12 + \frac{1}{8{,}800} [- 16\,(- 10 \times 3^3) + 2{,}160\,(- 10 \times 3 + 8)] = 7.09 \text{ kips}$$

In Fig. 29, point C is selected as the origin. From the formula of deflection in Table 4(*b*) for the boundary condition $\delta_A = 0$, substitute C for O, B for A, D for N, 5 ft for l_A, and 4 ft for x; thus:

$$\delta_D = \frac{1}{6\,E\,I \times 5} (- 4\, M'''_{CB,B} + 5\, M'''_{CD,D})$$

$$\delta_D = \frac{1}{30\,E\,I} \{- 4\,[10 \times 2^3 - (7.09 - 2)\,5^3 + 3 \times 8 \times 5^2] + 5\,(2 \times 4^3 - 3 \times 8 \times 4^2)\} = - \frac{291}{6\,E\,I} \text{ ft}$$

in which E is in kips per square foot, and I is in feet^4.

The writer is grateful to Mr. Floris, Professor Moorman, Professor McCormick, and Mr. Conwell for their discussions of this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

VARIATION OF COEFFICIENTS OF SIMULTANEOUS LINEAR EQUATIONS

Discussion

BY T. F. HICKERSON, A. FLORIS, AND FREDERICK S. MERRITT

T. F. HICKERSON,[2] M. ASCE.—This interesting paper is presented elaborately and clearly, except in certain instances where the author generalizes rather abruptly while introducing a multiplicity of unfamiliar symbols.

Two sets of simultaneous equations will be solved independently with a twofold purpose—(1) to help elucidate inductively the application of the author's key formulas and (2) to make comparisons of the relative advantages claimed by the author. Of the so-called classical methods the writer would eliminate determinants and include only the method of successive elimination by addition or subtraction after the coefficients of each unknown, in turn, have been made equal by multiplication or division—as was done in Table 1.

The solution of the set of three equations (Eqs. 22) given in Table 10 is believed to be self-explanatory. Anticipating a later variation of the coefficients, the letters a, b, and c are used here in a manner similar to that in Table 4. It will be seen that the equations are not renumbered when multiplied, or divided by any quantity, because in effect they are unchanged. Thus,

$$4\,x + 3\,y + 2\,z = 28 = a \quad\text{(36a)}$$

is equivalent to

$$12\,x + 9\,y + 6\,z = 84 = 3\,a \quad\text{(36b)}$$

In Table 10, let the coefficients of z in (1), (2), and (3) (Eqs. 22) be reduced by 2, 3, and 5, respectively, as was done in Table 4. Then a, b, and c will be replaced by $(a - 2\,z')$, $(b - 3\,z')$, and $(c - 5\,z')$, respectively. Hence, z', the revised value of z, becomes

$$z' = -\frac{2}{7}(a + 2\,z') + \frac{1}{9}(b + 3\,z') + \frac{13}{63}(c + 5\,z') \quad\text{(37a)}$$

NOTE.—This paper by Bernard L. Weiner was published in October, 1947, *Proceedings*.

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Rearranging terms,

$$z' + \left[\frac{2}{7}(2) - \frac{1}{9}(3) - \frac{13}{63}(5)\right] z' = -\frac{2}{7}a + \frac{1}{9}b + \frac{13}{63}c \ldots\ldots (37b)$$

It must be seen that the right side of Eq. 37*b* represents z ($= 5$), the initial (unrevised) value of z; the expression in the brackets equals z'_d and is comparable

TABLE 10.—Work Sheet for Solution of Eqs. 22

Equation	x	y	z	Constant term
(1)	4	3	2	$= 28 = a$
(2)	1	4	3	$= 26 = b$
(3)	5	2	6	$= 49 = c$
(1)	12	9	6	$= 84 = 3a$
(2)	2	8	6	$= 52 = 2b$
(4)	7	7	0	$= 35 = 3a - c$
(5)	10	1	0	$= 32 = 3a - 2b$
(4)	1	1		$= 5 = \frac{3}{7}a - \frac{1}{7}c$
(6)	9	0		$= 27 = \frac{18}{7}a - 2b + \frac{1}{7}c$
(6)	1 (x)			$= [3] = \frac{2}{7}a - \frac{2}{9}b + \frac{1}{63}c$
(4)		1 (y)		$= [2] = \frac{1}{7}a + \frac{2}{9}b - \frac{10}{63}c$
(1)			1 (z)	$= [5] = -\frac{2}{7}a + \frac{1}{9}b + \frac{13}{63}c$

Check: $x + y + z = 10 = \frac{1}{7}a + \frac{1}{9}b + \frac{4}{63}c = \frac{28}{7} + \frac{26}{9} + \frac{4 \times 49}{63} = 10$

to x_d in Eq. 15*b*, the numerical value of which is $-\frac{50}{63}$, as given in Col. 12, line 9, Table 4. By factoring Eq. 37*b*, and solving for z', $z' = \frac{5}{1 + \frac{50}{63}}$ (see Eq. 21*a*) $= \frac{315}{13}$, as recorded in Col. 5, line 12, Table 4.

To determine the corresponding revised values of x (designated x') due to the variation in the three coefficients of z, $x' = \frac{2}{7}(a + 2z') - \frac{2}{9}(b + 3z') + \frac{1}{63}(c + 5z') = x - \left(-\frac{4}{7} + \frac{2}{3} - \frac{5}{63}\right)z' = 3 - \left(\frac{1}{63}\right)\left(\frac{315}{13}\right) = \frac{34}{13}$, as recorded in Col. 5, line 10, Table 4. Likewise, $y' = \frac{1}{7}(a + 2z') + \frac{2}{9}(b + 3z') - \frac{10}{63}(c + 5z') = y - \left(-\frac{2}{7} - \frac{2}{3} + \frac{50}{63}\right)z' = 2 - \left(-\frac{10}{63}\right)\left(\frac{315}{13}\right) = \frac{76}{13}$, as recorded in Col. 5, line 11, Table 4. The revision due to the variation of the coefficients of z alone is now complete, and the revised set of equations is given by Eqs. 23. The variation will not be continued as was done by the author in

Table 4. Instead of breaking the four equations (Eqs. 35) into a pair of sets with two unknowns each, as is indicated in Table 8, the classical method is applied for purposes of comparison (Table 11).

TABLE 11.—WORK SHEET FOR SOLUTION OF EQS. 35

Equation	w	x	y	z	Constant term
(1)	10	9	8	7	$= 114 = a$
(2)	9	6	5	4	$= 76 = b$
(3)	8	5	3	2	$= 53 = c$
(4)	7	4	2	1	$= 39 = f$
(1)	10	9	8	7	$= 114 = a$
(2)	15.75	10.5	8.75	7	$= 133 = 1.75\,b$
(3)	28.0	17.5	10.5	7	$= 185.5 = 3.5\,c$
(4)	49.0	28.0	14.0	7	$= 273.0 = 7.0\,f$
(5)	−5.75	−1.5	−0.75	0	$= -19.0 = a - 1.75\,b$
(6)	−12.25	−7.0	−1.75	0	$= -5.25 = 1.75\,b - 3.5\,c$
(7)	−21.0	−10.5	−3.5	0	$= -87.5 = 3.5\,c - 7.0\,f$
(5)	7.667	2.0			$= 25.333 = -1.333\,a + 2.333\,b$
(6)	7.0	4.0	1		$= 30.0 = -b + 2\,c$
(7)	6.0	3.0	1		$= 25.0 = -c + 2\,f$
(8)	0.667	−2.0	0		$= -4.667 = -1.333\,a + 3.333\,b - 2\,c$
(9)	1.0	1.0	0		$= 5.0 = -b + 3\,c - 2\,f$
(8)	0.333	−1.0			$= -2.333 = -0.667\,a + 1.667\,b - c$
(10)	1.333	0			$= 2.667 = -0.667\,a + 0.667\,b + 2\,c - 2\,f$
(10)	1 (w)				$= [2] = -0.5\,a + 0.5\,b + 1.5\,c - 1.5\,f$
(9)		1 (x)			$= [3] = 0.5\,a - 1.5\,b + 1.5\,c - 0.5\,f$
(7)			1 (y)		$= [4] = 1.5\,a + 1.5\,b - 14.5\,c + 12.5\,f$
(4)				1 (z)	$= [5] = -1.5\,a - 0.5\,b + 12.5\,c - 11.5\,f$

Check: $w + x + y + z = 14 = c - f = 53 - 39 = 14$

TABLE 12.—WORK SHEET FOR SOLUTION OF EQS. 35 AND THE REVISION —WHEN THE COEFFICIENTS OF w ARE VARIED

Equation	w		x	y	z	Constant
(1)	10	(8)	9	8	7	= 114
(2)	9	(7)	6	5	4	= 76
(3)	8	(6)	5	3	2	= 53
(4)	7	(5)	4	2	1	= 39
(1)	10	(8)	9	8	7	= 114
(2)	15.75	(12.25)	10.5	8.75	7	= 133
(3)	28.0	(21.0)	17.5	10.5	7	= 185.5
(4)	49.0	(35.0)	28.0	14.0	7	= 273.0
(5)	−5.75	(−4.25)	−1.5	−0.75	0	= − 19.0
(6)	−12.25	(−8.75)	−7.0	−1.75	0	= − 5.25
(7)	−21.0	(−14.0)	−10.5	−3.5	0	= − 87.5
(5)	7.667	(5.667)	2.0	1		= 25.333
(6)	7.0	(5.0)	4.0	1		= 30.0
(7)	6.0	(4.0)	3.0	1		= 25.0
(8)	0.667	(0.667)	−2.0	0		= − 4.667
(9)	1.0	(1.0)	1.0	0		= 5.0
(8)	0.333	(0.333)	−1.0			= − 2.333
(10)	1.333	(1.333)	0			= 2.667
(10)	1 (w)					= 2.0 [2.0]
(9)			1 (x)			= 3.0 [3.0]
(7)				1 (y)		= 4.0 [8.0]
(4)					1 (z)	= 5.0 [1.0]

Check: $26\,w + 24\,x + 18\,y + 14\,z = 282$ and $52 + 72 + 144 + 14 = 282$

Omitting a, b, c, and f would shorten the work 50% or more, hence at least one of the coefficients may be varied and the corresponding equations resolved before starting the computation of the conversion factors proposed by the author. This point is illustrated in Table 12, in which varied coefficients of w and values of the constants are shown in parentheses. If, however, the constants are to be varied while the coefficients remain fixed—that is, if the loading conditions are varied while the framework remains unchanged—it is advantageous to express the values of w, x, y, and z in terms of a, b, c, and f.

A. FLORIS,[3] ESQ.—The proposed method of solving simultaneous equations is a modification of the process of successive elimination of unknowns. This is clearly seen from the "Remarks" in Col. 7, Table 1. From a practical point of view the author's method is of value only if a numerical error has been made in the statical analysis, necessitating the change of one or several coefficients of the unknowns. Hence, it is of restricted usefulness in the design of structures, since a change in the shape will require changing all the coefficients of the unknowns, including the terms that depend on the loading.

The method presented is not readily adapted to the solution of ill-conditioned equations; that is, equations in which different values for the unknowns can be obtained that will satisfy the equations.

The following equations, given by T. S. Wilson,[4] are cited as examples:

$$5x + 7y + 6z + 5u = 23 \qquad (38a)$$

$$7x + 10y + 8z + 7u = 32 \qquad (38b)$$

$$6x + 8y + 10z + 9u = 33 \qquad (38c)$$

and

$$5x + 7y + 9z + 10u = 31 \qquad (38d)$$

The exact values of the unknowns in Eqs. 38 are

$$x = y = z = u = 1 \qquad (39)$$

There are other solutions besides Eq. 39, however, as may be demonstrated by substituting the following three sets of values, successively, in Eqs. 38:

x	y	z	u
14.6	−7.2	−2.5	3.1
2.36	0.18	0.65	1.21
1.136	0.918	0.965	1.021

The result is three additional values for the right-hand side of Eq. 39, as follows:

Eq. 38a	Eq. 38b	Eq. 38c	Eq. 38d
23.1	31.9	32.9	31.1
23.01	31.99	32.99	31.01
23.001	31.999	32.999	31.001

Similar cases of ill-conditioned equations also can be found among those amenable to iteration.

[3] Los Angeles, Calif.

[4] "The Escalator Method in Engineering Vibration Problems," by Joseph Morris, John Wiley & Sons, Inc., New York, N. Y., 1947, p. 42.

Frederick S. Merritt,[5] Assoc. M. ASCE.—The writer has used the method of variation of coefficients on several occasions for solving the simultaneous linear equations arising in engineering practice. As a result of this experience, he has reached the conclusion that the method is especially well adapted for the purpose.

In the linear equations encountered in structural engineering, for example, the coefficients of the unknowns on the left side of the equation are generally functions of the dimensions of the structure and the constant terms on the right side are usually functions of the magnitude and position of the load. Classical methods of solution may readily be applied when the coefficients and the constants are unchanged, or even when the constants have several sets of values. However, when any of the coefficients are changed, say, because of change in the dimensions of a part of the structure, classical methods leave no recourse other than to re-solve the equations. On the other hand, the method of variation of coefficients makes it possible to evaluate, directly, the effect of a change in the coefficients.

Correction for *Transactions:* In October, 1947, *Proceedings*, on page 1224, Table 3, Col. 7, line 6, change "$b_3\,b$" to "$c_2\,b$".

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DISCUSSIONS

CLASSIFICATION AND IDENTIFICATION OF SOILS

Discussion

BY E. W. LANE, AND B. K. HOUGH

E. W. LANE,[20] M. ASCE.—It is believed that engineers working in soils would be glad to know of the size classification recently proposed by a Sub-Committee on Sediment Terminology of the American Geophysical Union (55)[20a] for use in work in sediments as given in Tables 15 and 16. The addition of a discussion of this classification to Professor Casagrande's excellent paper, therefore, seems justified.

TABLE 15.—SCALE OF LARGE PARTICLE SIZES, IN METRIC AND ENGLISH UNITS

Class name (1)	Metric unit (mm) (2)	English unit (in.) (3)
Very large boulders	4,096–2,048	160–80
Large boulders....	2,048–1,024	80–40
Medium boulders..	1,024–512	40–20
Small boulders....	512–256	20–10
Large cobbles.....	256–128	10–5
Small cobbles.....	128–64	5–2.5
Very coarse gravel.	64–32	2.5–1.3
Coarse gravel.....	32–16	1.3–0.6
Medium gravel....	16–8	0.6–0.3
Fine gravel.......	8–4	0.3–0.16
Very fine gravel...	4–2	0.16–0.08

TABLE 16.—SCALE OF SMALL PARTICLE SIZES

Class name (1)	METRIC UNITS (Mm) (2)	METRIC UNITS (Microns) (3)
Very coarse sand..	2.000 –1.000	2,000–1,000
Coarse sand......	1.000 –0.500	1,000–500
Medium sand.....	0.500 –0.250	500–250
Fine sand........	0.250 –0.125	250–125
Very fine sand....	0.125 –0.062	125–62
Coarse silt.......	0.062 –0.031	62–31
Medium silt......	0.031 –0.016	31–16
Fine silt.........	0.016 –0.008	16–8
Very fine silt.....	0.008 –0.004	8–4
Coarse clay size...	0.004 –0.0020	4–2
Medium clay size.	0.0020–0.0010	2–1
Fine clay size.....	0.0010–0.0005	1–0.5
Very fine clay size.	0.0005–0.00024	0.5–0.24

NOTE.—This paper by Arthur Casagrande appeared in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by Ralph E. Fadum; October, 1947, by James H. Stratton, and Donald J. Belcher; November, 1947, by J. A. Haine and J. W. Hilf, and Jacob Feld; January, 1948, by Kenneth S. Lane, George F. Sowers, René S. Pulido y Morales, Raymond F. Dawson, and D. F. Glynn; and March, 1948, by L. F. Cooling, A. W. Skempton, and R. Glossop, Milton Vargas, Donald M. Burmister, and M. G. Spangler.

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[20a] Numerals in parentheses, thus: (55), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

The classification is practically a modification of the classification (56) of C. K. Wentworth substituting the term gravel for his terms granule and pebble, subdividing his silt and clay sizes according to the widely used classifications proposed by J. A. Udden (57), and adding subdivisions of Mr. Wentworth's boulder classification. Most of the Wentworth-Udden classification has been widely used among geologists for many years and has been adopted by engineers in many sediment studies, but the granule and pebble classifications were not generally accepted.

The purpose of the Sub-Committee was to obtain agreement among engineers and geologists on a large number of terms in the sediment field, on which there was considerable difference of usage, including that of particle size. The Sub-Committee was composed of engineers and geologists, the engineers being G. H. Matthes, Hon. M. ASCE, L. G. Straub and the late G. C. Dobson, Members, ASCE, C. B. Brown, Assoc. M. ASCE, and the writer. The other members were equally well-known geologists.

The principal advantages of the proposed classification are believed to be as follows: (1) It gives exact subdivisions over the full range of particle sizes of sedimentary materials; (2) the terms used are common ones, and are applied to the range of sizes in common usage; and (3) the lower limit of each size is exactly one half of the upper limit, thus making the division points easy to remember and causing them to plot equally spaced on the semilogarithmic diagrams so widely used to indicate particle size distribution.

This equal size of classes is also a great advantage in the use of statistical methods in treating size ranges. Since the sizes of most sediment samples tend to form straight lines when plotted on logarithmic probability paper, the standard deviation of the logarithm of the sizes indicates the spread of the particle sizes. The composition of a sample can thus be closely represented by two numbers, the median (or geometric mean) and the standard deviation of the logarithm of the size. With these numbers, the composition of a large number of samples can be represented more conveniently than with a large number of size composition curves. It is sometimes possible to analyze a large mass of data by this method in illuminating ways which would be impossible by using the curves. This method of analysis was largely introduced into sedimentary science over a decade ago by W. C. Krumbein and has been widely used. It has more recently been independently introduced into soil mechanics by Donald M. Burmister, Assoc. M. ASCE.

B. K. Hough, Jr.,[21] Assoc. M. ASCE.—One of the many controversial aspects of this significant paper is the distinction between, and the relative importance of, the subject processes of soil classification and soil identification. It is noted that the author gives most attention to classification of soils, said classification being based on soil characteristics which are of various but generally limited interest, and which generally do not include identification. The question is raised herein as to whether identification may not be of major importance in all applications and the basis for more reliable classification and indication of probable soil behavior under service conditions.

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It is suggested that such terms as clay and silt, for example, are inadequate for true identification, although they may be sufficient in some applications as classifications. To designate a soil as clay, for instance, is scarcely more satisfactory than to designate one of a number of unidentified construction materials as wood. The differences between clay minerals are as basic and, in some respects, may be as important as the differences between oak and balsa, or pine and cedar. Not only mineralogical, but also chemical distinctions of great importance, notably base exchange capacity (BEC) and type of exchangeable ion, are in some cases of major interest.

Many engineers would agree that identification of soils (especially fine-grained soils) on the basis of their fundamental properties, mineral type, and composition in particular, is sound and desirable in principle but so difficult in practical application as to be of very limited utility, especially in the field. This rather general view may be influenced by misconception of the difficulty of such identification and insufficient knowledge of its practical value. It may now be the case that the practical value can be demonstrated and that identification methods have been simplified to such an extent that in the near future there will be a trend in engineering toward soil identification rather than toward the development of more complex empirical classification systems.

Mineral identification for many years was based on optical methods, which required highly specialized training and experience, usually in the field of geology or physics. For engineering applications this type of analysis has never seemed to be necessary and this may still be true although the increasing evidence of the value of such analysis might justify it. Other methods are available, however, such as the differential thermal analysis method, which can be simplified in application to almost any reasonable degree. Equipment for field use was developed during World War II by the United States Department of Agriculture and is now available for engineering purposes. This equipment can be operated quite satisfactorily by engineers in the junior professional categories. More elaborate equipment for analysis by the same method may be obtained and utilized in soils engineering laboratories without employment of specialists in this field.

The chemical analysis of a soil can also be organized in any reasonably well-equipped laboratory so that it may be conducted by technicians with minimum technical training, and in fact it is now on this basis in many agricultural laboratories. Equipment for field use is also available for certain types of chemical determinations.

In most, if not all soils engineering laboratories, the mechanical analysis is now a standard test and the data from such tests are considered essential in many systems of soil classification. It may well be, however, that in fine-grained soils specific surface is a more significant measurement than gradation or particle diameter, particularly when the latter are determined by such methods as the analysis of sedimentation by the specific gravity hydrometer. This observation is made as an extension of the foregoing comments on chemical analysis, inasmuch as specific surface, although in itself a physical characteristic, is usually indicative of chemical activity. In this respect it is fundamentally preferable to grain-size measurements as the basis for estimating such

soil characteristics as permeability, capillarity, plasticity, stability, and many other features which are of engineering interest. Data from hydrometer tests are of little use in such determinations and, in fact, are often accorded no more attention by engineers than a cursory examination despite the considerable cost of performing such tests.

The practical value of the identification of the clay minerals has been more clearly established in recent years. The swelling propensites of the montmorillonite group are generally recognized, although it is still not common practice to make an analysis of mineral types in cases where swelling is of particular importance. Many other correllations between mineral type and engineering behavior may be possible as studies in this area are undertaken. During recent soil tests under the writer's supervision, for example, standard Proctor compaction tests on an unknown clayey soil indicated an optimum moisture content of approximately 30% (dry weight basis). This value considerably exceeded that for any other soil tested by the writer in the course of many years of practical testing experience. Mineralogical analysis revealed the predominant clay mineral to be illite, which with its distinctive potassium bonding may well have characteristics that differ appreciably from those of the more common kaolinite group.

With reference to determination of the type of exchangeable ion, an almost classic example of the practical value of this soil characteristic is the treatment of the Treasure Island (Calif.) Lagoon lining (58) with ocean water to reduce its permeability. None of the soil classification systems described by the author would have indicated the feasibility of this eminently practical and effective treatment. Other examples of the practical importance of chemical reactions in soil in the field of highway engineering are almost too numerous to cite.

In conclusion, it is recommended that efforts be made by engineers to identify soils on the basis of fundamental properties—specifically, mineral type, exchangeable ions, BEC, and specific surface—and that thereafter soils be classified for various purposes. The basic identification will then be valid, no matter what the application or use of the soil may be, and might well serve as the common denominator for the many diversified interests now engaged in soil study and research.

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